Conic Sections in Context

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Abstract

This project is a discovery-based, multi-sensory unit composed of a series of lessons designed to teach high school students about conic sections. This four-week unit focuses on developing students' abilities to identify and/or create mathematical rules from tangible patterns. Its primary purpose is to examine conic sections and create connections between the geometric and algebraic definitions. As well, this unit is designed to challenge students to discover modern day applications of conic sections.

Activities of this unit include: computer-based explorations of conic sections, discussions of the etymology of each conic section, construction of each conic section using rope and sidewalk chalk, discovery of the standard formula for each conic section, and individual and group presentations on artistic creations and modern day applications of conic sections. This unit is designed to be presented in the second semester of an Algebra II course. Throughout this unit, students work in a number of environments to meet the need of each learning style. As well, students reflect daily in the form of a minute paper. These daily reflections facilitate an ongoing assessment of classroom dynamics.

Having been implemented at Los Alamos High School in March of 2011, assessment of student success and unit plan effectiveness included: samples of students' work, comparison of students' pretest & post-test results, and a review of student and colleague reflections, personal progress analysis, and unit evaluation surveys. Overall, assessment data supports the conclusion that this unit successfully fulfilled the target goals. In addition, assessment data suggests the following revisions of the plan: starting the unit with a webquest to establish relevancy, reorganizing the unit to address one conic section each lesson using a variety of approaches, and differentiating assessment.

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Introduction

Over 2000 years ago, conic sections became a topic of interest for mathematicians. In 350 BCE, Menæchmus discovered conic sections while trying to solve the Delian problem (doubling the volume of a given cube). A little over a hundred years later Apollonius of Perga, known as the "Great Geometer," systematically studied the conic sections and wrote eight books on the topic. For the next two millennia mathematicians continued to study the conic sections; yet it wasn't until the 14^{th} and 15^{th} centuries that their usefulness was fully uncovered. Franciscus Maurolycus (1494-1575), Johannes Kepler (1571-1630), Blaise Pascal (1623-1662), and René Descartes (1596-1650) all used conic sections to study various physical and astronomical problems[1]. Now in the 21^{st} century, high school students are studying the same set of curves.

My Dilemma

In my experience, however, conic sections are the most challenging topic addressed in the traditional Algebra 2 class, perhaps because of the difficulty in comprehending applications. As stated by one of my former students, "I just don't understand why we have to learn about this!"

Nature of the Problem

This perceived lack of relevancy frustrates and annoys the typical teenager. High school students rarely see the value of developing the critical thinking skills inherent in math and science. The naked theory behind these disciplines does not capture their interest; rather, young people are eager to know the how and why of the problems with which they grapple.

In a traditional math class, this reality is ignored—not for lack of awareness, though that is little comfort to a typical teenager. In most math books, the topic's relevancy is tackled either as a short introduction to the lesson or within a series of word problems at the end of the lesson. On occasion, historical sidebars are also included.

This failure to demonstrate relevancy is compounded by how a traditional math lesson is often conducted. For example, at Los Alamos High School, conic sections are introduced to students in the second semester of Algebra 2, a college preparatory class which follows Algebra 1 and Geometry. The primary text we use is an Algebra 2 textbook published by Glencoe/McGraw-Hill. Within the conic sections chapter, each conic section is addressed as a discrete lesson followed by extensive exercises related to model examples. Though the textbook includes hands-on activities and calculator explorations as supplements, the text is designed such that the primary mode of instruction is teacher-led.

My Solution

Thus, I was motivated to create a unit addressing the challenges stated above, one that integrated explicit relevancy and focused on student-centered instruction. Though trying to rewrite the entire Algebra 2 curriculum was an option, it was not a realistic option at that time. Instead, I

[1] Cajori, Florian. *A History of Mathematics*. London: Macmillan and, 1950. Print.

choose the one unit my students had universally hated, conic sections, and decided to rewrite it. As a fan of alliteration, I dubbed the unit “Conic Sections in Context” to emphasize the unit’s focus on relevancy and student-center instruction. Though the same core concepts are addressed in the “Conic Sections in Context” unit as are addressed in the textbook, the overall goal was for students to be actively engaged in the process of discovery, reflection, and creation. Thus, I began the task of researching lesson plans that would recursively spiral around these key concepts in imaginative ways and created a unit plan to integrate these lesson plans into a unique and cohesive unit with ample time allotted for reflection and synthesis.

Methodology

After defining the problem I wanted to address and how I wanted to address it, I began the task of seeking out information and materials that demonstrated how others had attempted to instruct students in the finer points of conic sections.

Compiling Sources

Both the New Mexico Institute of Mining and Technology’s online catalog (LIBROS) and the Internet were invaluable to me as I reviewed others’ approaches. Using LIBROS, I was able to access over a dozen articles from past volumes of *Mathematics Teacher*, a professional magazine published by the National Council of Teachers of Mathematics (NCTM) that “focuses on classroom activities and strategies for grades 8-14, deepening mathematical understanding, and linking research to practice.”[2] Topics of these articles ranged from the holistic “Folded Paper, Dynamic Geometry, and Proof: A Three-Tier Approach to the Conics” to the rigorous “A Direct Derivation of the Equations of the Conic Sections.” Discouragingly, the vast majority of these articles focus on topics found in Advanced Geometry and Pre-Calculus classes and are not level-appropriate for an Algebra 2 class.

In my continued search for level-appropriate material, I explored multiple resources on the Internet. The online resources I reviewed can be divided into three categories: online lesson plan repositories, teacher webpages and classroom tools. Of the online lesson plan repositories I use, I find NCTM’s *Illuminations* website the most worthwhile. Lessons are designed by teachers who have attended NCTM’s Illuminations Summer Institute, and always include learning objectives, materials, a detailed instructional plan, questions for students, assessment options, extensions, teacher reflection questions, and the NCTM Standards and Expectations covered by the particular lesson. In spite of a collection of over 600 lesson plans, I was surprised to find only two that related to conic sections: “Cutting Conics” and “Human Conics.” Happily, both these lessons were pivotal in the development of the “Conic Sections in Context” unit as will be discussed shortly.

[2] "High School Resources." *National Council of Teachers of Mathematics*. Web. 03 Apr. 2011. <http://www.nctm.org/resources/high.aspx>.

Though initially discouraged by the scarcity of level-appropriate material, I was encouraged when I began investigating teacher webpages and found two that were especially helpful. I sought out the teacher webpage of Ms. Leticia García de Espinosa of The American School Foundation of Guadalajara after viewing Youtube videos on conic sections. Her students had posted a number of videos regarding applications of conic sections in and around Guadalajara, Mexico. Her teacher page was also my first introduction to using art to reinforce and evaluate mathematical skills and knowledge. Shortly thereafter, I also discovered the teacher webpage of Rick Villano of Foothill Technology High School in Ventura, CA. Like Ms. Espinosa's, Mr. Villano's teacher webpage contained a bounty of resources, including assessments that incorporated art.

Hoping to integrate technology into the unit as well as art, I also reviewed a number of classroom tools. The three tools I found most interesting were: the Microsoft Word 2007 Math Add-In, Texas Instruments Conic Graphing Application, and Google Docs. The Microsoft Word 2007 Math Add-In is a free add-in available to Microsoft Word 2007 users. This add-in acts as a computer algebra system (CAS) and can turn Word into a graphing calculator. Unfortunately, the computer pods of Los Alamos High School use Microsoft Word 2002 and budget constraints prevent purchasing the necessary upgrade. At the individual level Texas Instruments' TI-83s and TI-84s have a conic graphing application that allows users to graph the four conic sections with ease, but while some TI-83s and TI-84s are available within my classroom, not all students own one. Rescue came in the form of Google Docs, because while Google Docs does not have a math equation editor, it does permit multiple people to edit a single document in realtime. A feature I found quite useful since I planned to incorporate writing into the unit.

Thus, due to a shortage of level-appropriate material (excluding traditional textbooks) and the financial constraints of my school, I acutely felt the need for the "Conic Sections in Context" unit. With the motley collection of resources I gathered, I focused my attention on creating a unit that emphasized relevancy and student-centered instruction while using widely available materials.

The Creation of a Unique Unit Plan

Wanting students to actively engage from the beginning, I structured the first lesson around the NCTM's *Illumination* lesson "Cutting Conics." I chose this interactive, online tool because it gives students the ability to see the 2-dimensional representation of a conic section while simultaneously viewing the 3-dimensional representation of a cone being sliced by a plane. As the first lesson of the unit, I also had students complete a pretest and introduced the concept of a minute paper, a writing to learn technique designed to identify points of confusion as well as help students retain information better. I followed this lesson with a cross-curricular lesson using etymology to reinforce the concepts of the first lesson, an idea I borrowed from "Word Histories: Melding Mathematics and Meanings" in the November 2000 issue of *Mathematics Teacher*. The goal of this lesson was to connect students' understanding of circles, ellipses, hyperbolas, and parabolas with other words students may have prior knowledge of in English—

such as circumnavigate, eclipse, hyperactive, and parasite—and in so doing reinforce their meaningfulness and relevance.

Having introduced the concept of conic sections as figures obtained from the intersection of a plane and a right double cone, I wanted the next set of lessons to delve into the geometric and algebraic properties of conic sections. The NCTM's *Illumination* lesson "Human Conics" brilliantly introduces the geometric properties of conic sections and allows mathematical principles to stretch beyond the confines of the classroom. Unfortunately the algebraic properties of conic sections are not so easily connected. In the end, I adapted a lesson from Course 4 of the *Core-Plus Mathematics* textbook, a student-centered text that introduces mathematical concepts through realistic problems and applications. As this lesson demands that students recognize patterns and fill in conceptual gaps, I dubbed the lesson "Clozing the Gap", a reference to an English as a Second Language (ESL) activity where cloze exercises (also known as gap fills) are used to develop fluency and accuracy.

Having established the conceptual, geometric, and algebraic definitions of conic sections with these four lessons, the remaining lessons were designed to focus on reviewing these definitions while encouraging students to discover various applications of conic sections. I began the latter half of the unit with a review lesson. I designed this lesson as a group activity where students summarize, explain, and present each of the three definitions of a conic section to their classmates and then create possible test questions. This peer-to-peer instruction solidifies key concepts and encourages mathematical communication. I followed this lesson with two project-based lessons, "Creating with Conic Sections" and "Conic Sections in Context Project". As mentioned earlier, I was inspired to integrate art into this unit after discovering Ms. Espinosa's and Mr. Villano's teacher webpages and reading "Draw It, Write It, Do It" in the October 2005 issue of *Mathematics Teacher*. The "Draw It, Write It, Do It" article inspired the structure of the lesson while I used the artwork posted on Ms. Espinosa's website as examples of conic art and the project description posted on Mr. Villano's to provide a clear description of expectations. In addition to blending these resources to form the conic art project, I created the "Conic Sections in Context Project"—a webquest designed to answer the infamous question: "When are we ever going to use this?!?" Having never designed a webquest in the past, I researched other webquests and settled on the general format of introduction, task, process and resources. Using historical facts for the introduction, I wrote the task and process descriptions to focus students' attention on the applications of conic sections. Having reached the acme of the unit, I designed the last two lessons such that students end where they begin by retaking the pretest and synthesizing their minute papers into a single personal progress reflection.

As is evident, the "Conic Sections in Context" unit is a compilation of resources. Excluding the "Conic Section Pretest" and the "Conic Sections in Context Webquest", all student materials within this independent study are drawn from other sources and cited appropriately. Conversely, apart from the NCTM's *Illumination* lesson "Human Conics," I am the author of the unit plan with any use of other authors' images, text, or concepts cited appropriately. Consequently, the originality of this unit lies in how I wove these loosely related lessons into a unique and cohesive unit focused on students actively engaged in discovery, reflection, and creation.

Measuring Success

The success of my approach to conic sections is assessable in multiple ways. As discussed earlier, each lesson's structure includes time for the teacher and the students to reflect on the key concepts introduced in the lesson as well as how these concepts connect to those of earlier lessons. The goal of these reflections is three-fold: to improve self-awareness, to learn by analyzing experiences, and to reinforce understanding by constant revisiting of concepts[3]. In addition, each lesson is accompanied by a homework assignment that builds on the concepts introduced in the lesson, highlights key concepts, and encourages students to make connections between each lesson. Lastly, I designed the unit to begin and end with the same test for easy objective measurement of individual progress; students also complete a personal progress reflection and unit evaluation to measure subjective success.

Results & Conclusion

Implementation of this unit occurred March of 2011 at Los Alamos High School. An analysis of the work of the students and staff involved is conducted in the results chapter of this paper. An overall assessment of the project with regard to what worked, what didn't work, and how I plan to change the project when I implement it again in the future is addressed in the conclusion of this paper. The two chapters that precede the results and conclusion chapter are the unit plan and the associated student materials.

[3] Benek-Rivera, Joan. "By Teaching You Will Learn: Journals Facilitate Student and Faculty Learning." *MountainRise* 2.1 (2005): 82-91. Web. 22 Feb. 2011. <http://mountainrise.wcu.edu/index.php/MtnRise/article/viewFile/50/82>.

Unit Plan

Course: 9^{th}-12^{th} Grade Mathematics (Algebra II)

Unit Title: Conic Sections in Context

Goal: The goal of this unit is for students to be able to identify applications of conic sections by examining the properties of conic sections and creating connections between the geometric and algebraic definitions.

Standards & Benchmarks:

NM Standards for Grade 9-12 Mathematics

- 9-12.G.1.5 Use definitions in making logical arguments.
- 9-12.G.2.2 Determine the midpoint and distance between two points within a coordinate system and relate these ideas to geometric figures in the plane (e.g., find the center of a circle given the two points of a diameter of the circle).
- 9-12.G.2.3 Use basic geometric ideas (e.g., the Pythagorean Theorem, area and perimeter) in the context of the Cartesian coordinate plane (e.g., calculate the perimeter of a rectangle with integer coordinates and with sides parallel to the coordinate axes, and of a rectangle with sides not parallel).
- 9-12.G.3.2 Sketch a planar figure that is the result of given transformations (i.e., translation, reflection, rotation, and/or dilation).
- 9-12.G.3.3 Identify similarity in terms of transformations.
- 9-12.AX.2.3 Graph, interpret, and find the equations for conic sections with axes parallel to the coordinate axes, and apply them to contextual situations.

NM Benchmarks for Grade 9-12 Mathematics

- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
- Specify locations and describe spatial relationships using coordinate geometry and other representational systems.
- Apply transformations and use symmetry to analyze mathematical situations.

Essential Question:

"Whether created with compass and straight-edge, Moiré patterns, iterative paper-folding, or computer modeling, why have conic sections remained relevant for over 2000 years? How does this change the way you approach math?"

Unit Concepts and Generalizations:

Conic Sections:

- are planar figures obtained from the intersection of a plane and a right double cone.
- are a set (or locus) of points satisfying a specified distance condition.
- are quadratic relations of the form: $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$.

Performance Objectives:

As a result of this unit, students will **KNOW**

- the basic properties of circles, ellipses, parabolas, and hyperbolas
- the standard equation of circles, ellipses, parabolas, and hyperbolas
- some applications of circles, ellipses, parabolas, and hyperbolas

As a result of this unit, students will **UNDERSTAND that**

- Circles, ellipses, parabolas, and hyperbolas can be defined geometrically and algebraically
- Circles, ellipses, parabolas, and hyperbolas can be translated within the Cartesian plane

As a result of this unit, students **WILL BE ABLE TO**

- Describe each of the conic sections in terms of the intersection of a plane and a double cone
- Sketch each conic section and label relevant points
- Present on modern applications of circles, ellipses, parabolas, and hyperbolas
- Create artistic drawings using circles, ellipses, parabolas, and hyperbolas
- Work cooperatively in small groups to solve problems

Instructional Strategies:

- Pre- and Post-test
- Brainstorming
- Graphic organizers
- Gardner's multiple intelligences
- Sternberg's triarchic intelligences
- Visual, aural, and kinesthetic learning
- Think-Pair-Share
- Metacognitive thinking
- On-going assessment (exit cards / journals / reflections)
- Student-generated test questions
- Rubric assessment

Supporting Materials:

- Computers with Internet Access and Presentation Software
- The Words of Mathematics: An Etymological Dictionary of Mathematical Terms Used in English by Steven Schwartzman
- Glencoe Algebra II (ISBN 0-07-875624-3)
- TI 83/84 Graphing Calculators
- Sidewalk Chalk
- Lightweight rope (about 10-12 feet per group)
- Patty Paper
- Graph Paper (standard & Moiré patterned)
- Index cards for on-going assessment
- Large sheets of paper for small-group brainstorming
- Post board for review activity
- Music to accompany "work time"
- NCTM Standard Rubric for Mathematics

Unit Summary: This four-week unit focuses on developing students' abilities to identify and/or create mathematical rules from tangible patterns. Its primary purpose is to examine conic sections and create connections between the geometric and algebraic definitions. As well, students will be challenged to discover modern day applications of conic sections. Activities of this unit include: computer-based explorations of conic sections, discussions of the etymology of each conic section, construction of each conic section using rope and sidewalk chalk, discovery of the standard formula for each conic section, and individual and group presentations on artistic creations and modern day applications of conic sections. This unit is designed to be presented in the second semester of an Algebra II course. Throughout this unit, students will work in a number of environments to meet the need of each learning style and reflect daily to permit the teacher a comprehensive assessment of classroom dynamics.

"Conic Sections in Context" Unit Overview

Lesson Title & Timeline	Learning Objectives	Assessment
Discovering Conic Sections through Technology *1 class period*	Students will: • Complete the conic section pre-test • Explore how each conic sections is formed using a computer-based application • Compare and contrast the properties of at least two conic sections	Conic sections pre-test In-class activity sheet In-class Journal entry/Minute paper Homework: Compare and contrast the properties of at least two conic sections.
Finding Meaning in the words of Mathematics *1 class period*	Students will: • Identify the roots of the mathematical terms for each conic section • Research the etymology of each root • Brainstorm related common English roots • Analyze how the etymology of each root is related to how each conic section is formed when a plane intersects a double right cone.	In-class posters In-class exit card/Minute paper Homework: Summary of the day's activities for a hypothetically absent student
Graphing with Sidewalk Chalk & Rope *2 class periods*	Students will: • Define conic sections as a locus of points • Apply locus definitions to draw conic sections • Summarize the relationship between the focus-focus/focus-directrix for each conic section	In-class activity sheet In-class Journal entry/Minute paper Homework: Exercises related to Moiré patterns

Clozing the Gap *2 class periods*	Students will: • Fold paper to form a conic section • Use the distance formula and the completing the square method to derive the general formula for each conic section • Apply findings and match various equations to the appropriate conic section	In-class activity sheet In-class exit card/Minute paper Homework: Exercises related to identifying & sketching a conic section given its equation and vice versa
Relating the Many Definitions of Conic Sections *1-2 class periods*	Students will: • Identify the key concepts regarding conic sections • Create a summative chart in a small-group • Recommend 2-3 test questions for the unit test	Summative chart Suggested test questions Homework: Create a solution set for the class created test questions
Creating with Conic Sections *1-2 class periods*	Students will: • Make a drawing that only includes lines, parabolas, circles, ellipses, and hyperbolas • Write the equations used to create the drawings • Recreate another student's drawing given his/her equations	Student drawings In-class activity sheet In-class Journal entry/Minute paper Homework: Exercises related to writing equations based on various drawings
Conic Sections in Context Project *2-4 class periods*	Students will: • Research real-world applications of conic sections • Summarize the related mathematical principles • Present findings to the class	Student PowerPoint presentations Homework: 2-3 page paper summarizing various applications of conic sections based on class presentations
Unit Test Review *1 class period*	Students will: • Discuss unit generalizations • Review individual solution sets to the class created test questions in a small group • Reflect on individual progress throughout the unit	Individual solution sets to class created test questions Homework: Evaluate each of the unit activities.
Unit Test *1 class period*	Students will: • Complete the conic sections post-test • Submit unit evaluations	Conic sections post-test Unit Evaluations

"Conic Sections in Context" Unit Description and Teacher Commentary

Lesson 1	Discovering Conic Sections through Technology[1]	(*1 class period*)

LESSON SEQUENCE AND DESCRIPTION	TEACHER COMMENTARY
Assessment of Prior Knowledge (10 minutes) Hand out the Conic Sections Pretest. Working individually, have students complete the pretest. When the majority of the class has completed the pretest, ask students to come to a stopping point & turn in their pretests. Tell students not to worry if they could answer few if any of the pretest questions. Over the next few weeks, the class will be exploring each of these topics in detail. *Engage (5 minutes)* Working in partners, have students access the Conic Section Explorer tool at: http://illuminations.nctm.org/ActivityDetail.aspx?id=195. As a class, briefly experiment with how varying each parameter changes the two views. Ask students to write down a hypothesis as to what shapes will be formed as the parameters are changed. *Explain (15 minutes)* Review the questions posed under the Exploration tab. Instruct students to respond to these questions in their notebooks during their exploration. Review & clarify directions before students begin. Allot 15 minutes for students to complete the investigation. *Elaborate (10-15 minutes)* Have students switch partners and review discoveries. After a few minutes, ask students to regroup and compare & share as a class. Post the "Questions for Students". "How can a cone be cut to create a circle? How can a cone be cut to create an ellipse? How can a cone be cut to create a hyperbola? How can a cone be cut to create a parabola? How are circles and ellipses related? How are hyperbolas and parabolas related?" As a class, summarize the discoveries of the day & post on the whiteboard/overhead.	If students are struggling, ask guiding questions like: "What happens if you change the slope of the plane? What happens if you change the location of the plane and place it closer to the origin? What happens if you change the value of b?" Circles are formed when the plane is parallel to the cone (e.g. m=0). Likewise, an ellipse is formed when the slope of plane is greater than 0, but less than the slant of the cone. Conversely, hyperbolas are formed when the slope of the plane is greater than the slant of the cone and a parabola is formed when the slope of the plane & the slant of the cone are the same.

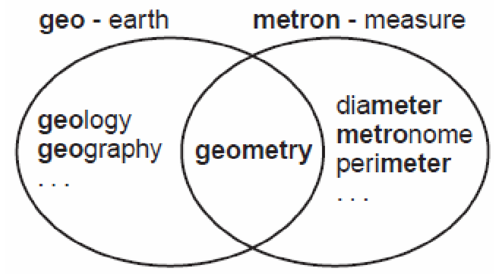
[1] Source for "Questions for Students" and guiding questions: Johanson, Terry. "Cutting Conics." *Illuminations*. National Council for Teachers of Mathematics. n.d. Web. 22 Dec. 2010.

Evaluate (5 minutes) Lastly, introduce the concept of a minute paper[2]. Instruct students to reflect on the activities of the day by answering the following questions: • What was the most surprising and/or instructive discovery you made in class today? • What unanswered question do you still have? Post homework assignment & remind students it is due next class. *Homework* Assign each student one of the six pairs of conic sections (circle/ellipse, circle/hyperbola, circle/parabola, ellipse/hyperbola, ellipse/parabola, and hyperbola/parabola). Each student is to summarize what they learned about the two conic sections in class and compare and contrast them. Emphasize that each student's work will be shared with others in the class. *Teacher Reflection* Did the online activity enhance student understanding in regards to conic sections? If so, how so? How did this lesson engage students of varying learning styles? What percentage of the time were students actively engaged in the learning process? How could this lesson be improved?	In regards to how various conics are related, circles and ellipses are both closed curves and increase in size as the plane moves away from the origin (e.g. b increases). Similarly, hyperbolas and parabolas are both open curves which extend to infinity. As well, the vertices of both hyperbolas and parabolas get rounder as the plane moves away from the origin. Note, however, hyperbolas approach their asymptotes where parabolas do not.

[2] Angelo, T.A., and Cross, K.P. *Classroom Assessment Techniques*, 2nd ed., Jossey-Bass, San Francisco, 1993, pp. 148-153

Lesson 2 Finding Meaning in the words of Mathematics[3] (*1 class period*)	
LESSON SEQUENCE AND DESCRIPTION	TEACHER COMMENTARY
Engage (10 minutes) As students enter class, group students according to which pair of conic section he/she summarized. Groups should be heterogeneous with a representative of each pair of conics present in each group. Students should compare and share their responses with those in their group and record their observations on a single piece of paper. If time permits, discuss conclusions as a class. When the task is complete, each group should turn in their homework as well as the group's summary.	By starting the lesson by reviewing the previous lesson's homework, the hope is for students to make connections between the two lessons as well as give students the opportunity to be "experts". Do not isolate students who fail to complete their homework. Rather encourage him/her to participate to the degree he/she is able and submit their comparisons the next day for reduced credit.
Explain (5 minutes) Post an image of multiple planes slicing a cone (or multiple cones). Ask students: "how are the words for each of these curves (e.g. ellipse, parabola and hyperbola) related to how these curves are formed?" Record student ideas on classroom whiteboard or overhead. Tell students today the lesson will focus on answering that question to help students make connections between the word origins and how conic sections are formed.	Images of multiple planes slicing a cone (or multiple cones) are available in Appendix B.
Elaborate (25-30 minutes) Post a Venn diagram with two circles intersecting. In the space where the circles intersect write the word "geometry". Ask students what two roots "geometry" can be split into. Give hints if needed. Write the first root above the circle on the right and the second root above the circle on the left. Write the definition of each root next to it (e.g. geo-earth and metron-measure). Then ask students for other words that contain the root "geo-" and "-metron". Record student responses in the appropriate circle. See below for an example.	As suggested in Rubenstein and Schwartz's "Word Histories: Melding Mathematics and Meanings," Aristotle built on the Pythagoreans concept to ellipses as a defect, hyperbole as an excess, and a parabole as alongside. With this in mind, an ellipse "falls short" as is evident in terms like elliptical, ellipsis,

[3] Source of lesson concept and images: Rubenstein, Rheta N., and Randy K. Schwartz. "Word Histories: Melding Mathematics and Meanings." *Mathematics Teacher* 93.8 (Nov 2000): 664-69. Print.



Divide students into three groups and assign each group to brainstorm words of similar etymology given the main words of hyperbola, parabola, or ellipse. If the class is large, assign the same term to multiple groups, but maintain group size to 3-4 students. Distribute dictionaries and Steven Schwartman's "The Words of Mathematics" as references. Instruct students that they have 10 minutes to find as many related words as they can. Pass out large pieces of butcher paper for students to summarize their findings.

Evaluate (5-10 minutes)
As each group completes their poster, post the student responses around the classroom & instruct students to walk around the classroom as if in a gallery and record their observations in their notebooks/journals.
If time permits, instruct the class to regroup and ask each group to present a 30 second mini-summary of their conclusions. Record the mini-summaries on the classroom whiteboard/overhead.

Reinforcing the concept of a minute paper, instruct students to reflect on the activities of the day by answering the following questions:

- Do you feel language and mathematical concepts are related? If so, how was that interdependence evident in today's lesson? If not, what characteristics make the two independent of each other?
- What unanswered question do you still have?

Post homework assignment & remind students it is due next class.

Homework
Instruct students to create a summary of the day's activities for a hypothetically absent student. Each summary should include a description of the activity as well as the conclusions drawn. Summaries will be graded on clarity of communication, evidence of mathematical connections or observations, and appropriate and accurate mathematical representations. Note, the National Council for Teacher of Mathematics' (NCTM's) Standards-Based Math Rubric may be adapted for this purpose.

or an eclipse; a hyperbola "goes to far" as is seen in terms like hyperbole, hyperactive, hypersonic, and hypertension; and a parabola "matches" or is parallel as is understood in terms like parable, paragraph, and parasite.

Teacher Reflection Did students' understanding of the etymology of the words *hyperbola*, *parabola*, and *ellipse* improve their "general knowledge of language and their mathematical fluency"? If so, how so? If not, what additional steps were needed to make the connection? Did this lesson successfully engage students with a particular interest in the arts and humanities? What percentage of the time were students actively engaged in the learning process? How could this lesson be improved?	

Lesson 3 Graphing with Sidewalk Chalk & Rope[4] (*2 class period*)	
LESSON SEQUENCE AND DESCRIPTION	TEACHER COMMENTARY
In preparation for the lesson, mark the midpoint of each rope with a permanent marker or a piece of tape. Day 1 (Circle & Ellipse) *Engage (5 minutes)* Remind students the last two lessons have focused on conic sections formed by slicing a double right cone. Tell students that today they will learn a second way to describe those same conic sections. Give each student a compass and the <u>Human Circle</u> activity sheet. *Explain (5-10 minutes)* Ask students: • What is the definition of a circle? [a set of points equidistant from a given point called the center] • What do the parts of the compass represent in this definition? [needle = center, pencil marks = circle, distance from needle to pencil point = radius] For a large group demonstration, replace the pencil in a compass with an overhead marker and demonstrate the use of the compass on a transparency. Emphasize the importance of not squeezing the compass so that the radius is maintained. Let students practice by having each of them draw a circle with a radius equal to the length of their index finger. Discuss how the construction is related to the locus definition. Remind students that the circle is just the locus of points, not its interior. *Elaborate (10 minutes)* Hand out the <u>Human Ellipse</u> activity sheet. Show students the <u>Ellipse Definition</u> overhead, covering the title, and ask what shape they see. If students say oval, explain that an oval and an ellipse may look alike, but the shapes we deal with in our study of conics are called ellipses. Indicate the foci and	Note, if this lesson is completed in a single block period, complete all classroom activities prior to exiting the classroom to work on the outdoor activities. All activity sheets, answer keys and overheads for this lesson may be found in the "Student Materials" chapter

[4] Lesson description and materials sourced verbatim from: Bush, Ellen. R.S. "Human Conics." *Illuminations*. National Council for Teachers of Mathematics. n.d. Web. 28 Dec. 2010.

simply state that these are called focal points or foci. Give students the definition of an ellipse. Point out that "foci" is the plural of focus. Use different colors to illustrate the definition by selecting points on the ellipse and drawing lines to the foci. Do not answer questions or engage students in discussion so that students may ponder the definition as they work outside. *Evaluate (20-30 minutes)* Before going outside, separate students into groups of three. Three students are needed for the ellipse and parabola, two to represent foci or directrix and one to draw. For the circle, only two students are required, the center and the chalk, but it is usually easier not to rearrange groups mid-activity. Explain to students that they will be working in groups to draw a circle, an ellipse, and a parabola [though the parabola will not be addressed until Day 2]. Each group will have one piece of chalk and one piece of rope. Do not give instructions or hints on how to draw the conics until students have had ample opportunity to experiment.	If students need further instruction: Fold the rope in half. One student puts the ends together, and holds them on the ground to be the center of the circle. The second student stretches the rope and puts the chalk in the bend at the midpoint. The second student then drags the chalk along the ground, while pulling the rope taut. Note that students figuring out the activity independently may not fold the rope. This is not a problem.
<u>Circle</u> Instruct students to draw a perfect circle using the chalk and the rope. If students need a hint, suggest that they consider themselves to be a human compass. <u>Ellipse</u> If students need hints, tell them that the fact that there are three people in the group is significant and to consider what they did to draw the circle. Once all groups have completed the construction of both the circle & ellipse, have students return to the classroom. If time permits, reorganize groups so each group contains only one representative from one of the original groups and have students compare and share their responses to the <u>Human Circle</u> and <u>Human Ellipse</u> activity sheet. Continuing with the concept of a minute paper, instruct students to reflect on the activities of the day by answering the following questions: • Did working outside help you better understand the locus definitions of a circle and an ellipse? If so, how so? If not, why not? • What unanswered question do you still have?	If students need further instruction: Two students are human foci, holding the ends of the rope at fixed points on the ground. These students should not hold the rope taut. The third student uses the chalk to pull the rope taut and sweeps out the locus of points. As students finish, ask them to consider and discuss the questions on the activity sheet.

Post homework assignment & remind students it is due next class. *Homework* Similar to Lesson 2, instruct students to create a summary of the day's activities for a hypothetically absent student. Each summary should include a description of the activity, the definition of a circle and an ellipse, and an explanation of how the drawing technique applies the definition. Summaries will be graded on clarity of communication, evidence of mathematical connections or observations, and appropriate and accurate mathematical representations. Note, the National Council for Teacher of Mathematics' (NCTM's) Standards-Based Math Rubric may be adapted for this purpose. Alternately, those with access to Glencoe <u>Algebra II</u> (ISBN 0-07-875624-3) 2005 edition, may instruct students to complete the Algebra Activity "Investigating Ellipses" on page 432. An image of this page may be found in the "Student Materials" chapter. *Teacher Reflection* How did students respond to the opportunity to work outside? To what extent were students able to do the activities without instruction? Can the circle activity be skipped for some of your students? Did the students understand the definitions before going outside? If not, did the outside activity clarify the definitions? What percentage of the time were students actively engaged in the learning process? How could this lesson be improved?	
Day 2 (Parabola & Hyperbola) *Engage (5-10 minutes)* Review the previous lesson by asking students how many students were actually needed to draw the circle? [two] What were the students' roles? [center of the circle and point on the circle] What did the rope represent? [the constant distance or radius] How many students were needed to draw an ellipse? [three] What were the students' roles? [center of the 2 foci and a point on the ellipse] What did the rope represent? [the constant sum of the distances from a point to each of the foci] Post the <u>Parabola Definition</u> overhead. *Explain (5-10 minutes)* In small groups, instruct students to brainstorm how a parabola	

could be constructed using chalk, a rope, and a stiff piece of cardstock or a right angle ruler. As a class, review each group's suggestions and discuss challenges and feasibility of each method. *Elaborate (20 minutes)* If students come up with a feasible method, encourage the students to move outside and test out their theories. For classes which struggle to find an appropriate method, gather students for an outdoor demonstration. Have one group of students demonstrate the parabola drawing as you direct them through the instructions: • Draw a focus approximately 2 feet from the directrix. This does not need to be precise. You are just looking for a distance that will allow students room to maneuver and will produce an easily recognizable parabola. • Assign roles to the three students in the group: F, D, and A. Student D will be responsible for the directrix and will need a right angle measure, such as cardstock or a right angle ruler, to approximate right angles. Student F will be responsible for the focus of the parabola. Student A will mark points on the parabola. • Assign each student a point on the rope. Student A is at the marked midpoint of the rope. At equal distances from her, measured by folding the rope, are F and D. • F should hold her point of the rope at the focus on the ground. D should place the right angle measure on the directrix and guide the rope along the side of the measure. She should move the card and rope along the directrix while A pulls the rope taut. When the rope is taut and perpendicular to the directrix, A should mark the point on the parabola. In the figure to the right, person D is correctly positioned perpendicular to the directrix, but person B is not. Students do not need to mark the congruent lengths on the ground, although some will naturally do this to clarify their thinking. • Students use the same rope length and repeat the procedure to draw a point on the other side of the parabola. Then, change lengths and repeat for a total of at least six points. Now challenge each group to complete their own constructions.	The following questions should be asked of students while they are still outside completing their constructions. These questions serve as the closure to the lesson, and should be used to ensure that all students have a conceptual understanding of the locus definitions of parabolas. • What is the shortest segment from the focus to the directrix? [the perpendicular segment that goes through the vertex] • What is the midpoint of this segment? [the vertex of the parabola] • Why is it important to keep the rope perpendicular to the directrix? [The distance between a point and a line is the perpendicular to the line.] • How can you find the vertex of the parabola using your rope, right angle measure, and group members? [Use your materials to find the perpendicular segment from the focus to the directrix. The vertex is the midpoint of the segment.]

Evaluate (10-15 minutes) When all groups have completed and/or observed a construction, have students return to the classroom. Ask students the following questions: • What effect does the length of the rope have on the shape of the conic? Is the rope ever too short? [A longer rope makes a larger conic. When the rope length is equal to the distance between the focus and directrix of the parabola or the distance between the foci of the ellipse, the conic can no longer be created.] • Why can you draw the circle with fewer people than the ellipse or the parabola? [The circle only has one fixed element. This question could lead to a more general discussion about the differences between the conics.] • Which conics can you draw as a continuous line, without picking up your chalk? [The circle and the ellipse were easy to draw as a continuous line. The parabola can also be drawn continuously, but it would be far more difficult because of the need to maintain the right angle to the directrix.] • How could you use paper and pencil to draw or verify conics? [Be open to all student suggestions and let them try to demonstrate as many as feasible. Some ideas may prove to be more difficult than they anticipate. Ideas for drawing may include taping the ends of a string to paper, measuring each point, or using multiple rulers. The definitions can be verified by using a ruler to measure distances between points on the conic and the foci or directrix.] • What conic have we failed to create using rope and chalk? [A hyperbola.] If time allows, post the Hyperbola Definition overhead. As a class, discuss reasons that the hyperbola would be difficult to draw using chalk and rope. Point out the fact that the hyperbola consists of 2 distinct branches. Otherwise, show students the "Constructing a Hyperbola animation" available at: http://mathdemos.gcsu.edu/mathdemos/conic_via_locus/ and discuss.	

Instruct students to reflect on the activities of the day by answering the following questions: • Did working outside help you better understand the locus definitions of a parabola? If so, how so? If not, why not? • What unanswered question do you still have? Post homework assignment & remind students it is due next class. *Homework* Those with access to Glencoe Algebra II (ISBN 0-07-875624-3) 2005 edition, may instruct students to complete the Algebra Activity "Conic Sections" on page 453-454. An image of this page may be found in the "Student Materials" chapter. Similarly, provide students with conic graph paper and ask student to experiment with how to create the various conic sections using the Moiré patterns. A sample set of exercises may be found in the "Student Materials" chapter. *Teacher Reflection* Did the students understand the definition of a parabola before going outside? If not, did the outside activity clarify the definition? Did you find it necessary to make adjustments while teaching the lesson? If so, what adjustments and were these adjustments effective? What percentage of the time were students actively engaged in the learning process? How could this lesson be improved?	

Lesson 4 Clozing the Gap (*2 class period*)

LESSON SEQUENCE AND DESCRIPTION	TEACHER COMMENTARY
Day 1 (Circles & Ellipses) *Engage (5-10 minutes)* Post the unit concepts & generalizations. Ask a student to read the unit concepts & generalizations aloud. With a partner or in a small group have students discuss how each of these definitions has been addressed thus far and which definition has yet to be discussed. Call on a few students from each pair (or small group) to share. Record student observations on the classroom whiteboard/overhead. *Explain (5 minutes)* Tell students this lesson will focus on the last unit concept & generalization to be addressed. Encourage students to recall the definition of a circle and an ellipse from the last lesson. Explain that by the end of the lesson, they will be able to describe any circle or ellipse given its equation and vice versa. *Elaborate (20-30 minutes)* Hand out the Circle Cloze the Gap activity sheet. Working in groups of 3-4 people, encourage students to complete the derivation of the standard form of the equation of a circle by completing the Cloze the Gap activity. Circulate and assist as needed. After the majority of the groups have completed the Circle Cloze the Gap activity sheet, rearrange group members so individuals from different groups can compare and share their results. As a class, address any misconceptions or commonly made errors. Next, have students return to their original groups and hand out the Ellipse Cloze the Gap activity sheet. Again, students will complete the derivation of the standard form of the equation of an ellipse by completing the Cloze the Gap activity. Circulate and assist as needed. As well, after the majority of the groups have completed the Ellipse Cloze the Gap activity sheet, rearrange group members so	This lesson assumes students have covered (and remember!) the distance formula and how to complete the square. If not, precede this lesson with a set of review exercises and/or a complete lesson. Suggested online tutorials for individual students, include: www.patrickjmt.com and www.khanacademy.org. When students are working on the Cloze the Gap activity and need help, reinforce correct methodology and ask questions rather than give answers when assisting. Ideally, the Circle Cloze the Gap activity sheet will provide a model for students on how to complete the derivation of an equation as well as build confidence before working with more complicated derivations.

individuals from different groups can compare and share their results. As a class, address any misconceptions or commonly made errors. *Evaluate (10 minutes)* Once all groups have had a chance to discuss, post an exercise similar to those assigned for homework. Ask students to following questions: How can you tell if this equation is a circle or an ellipse? What method would we use to rewrite this equation to make it easier to graph? What information does the standard form of a circle give us that makes it easier to graph? What information does the standard form of an ellipse give us that makes it easier to graph? Instruct students to reflect on the activities of the day by answering the following questions: • Did deriving the standard form of the equation of a circle and an ellipse help you better understand the definition of a circle and an ellipse? If so, how so? If not, why not? • What unanswered question do you still have? Post homework assignment & remind students it is due next class. *Homework* Instruct students to complete exercises related to identifying & sketching circles and ellipses given their equation and vice versa. A sample set of exercises may be found in the "Student Materials" chapter. *Teacher Reflection* How did students respond to desk work after multiple lessons using less traditional approaches? What percentage of the time were students actively engaged in the learning process? How could this lesson be improved?	
Day 2 (Parabola & Hyperbolas) *Engage (5-10 minutes)* Have students work with a partner (or small group) to review the previous night's homework.	This lesson assumes that students have worked extensively with algebraically manipulating quadratic equations in prior semesters/chapters.

Review & discuss as a class. Ask students which conic sections still need to be addressed algebraically. Post student responses. *Explain (5-10 minutes)* Post the locus definition of a parabola from "Graphing with Sidewalk Chalk & Rope", the vertex form of the equation of a parabola, and a quadratic of the form $ax^2 + bx + c = 0, a \neq 0$. Ask students how these definitions are related. Post student responses. *Elaborate (20-30 minutes)* Hand out the <u>Parabola Cloze the Gap</u> activity sheet. Working in groups of 3-4 people, encourage students to explore the multiple algebraic definitions of parabolas and make connections between what information can be easily gleaned from each definition. Circulate and assist as needed. After the majority of the groups have completed the <u>Parabola Cloze the Gap</u> activity sheet, rearrange group members so individuals from different groups can compare and share their results. As a class, address any misconceptions or commonly made errors. Next, have students return to their original groups and hand out the <u>Hyperbola Cloze the Gap</u> activity sheet. Students will now complete the derivation of the standard form of the equation of an hyperbola by completing the Cloze the Gap activity. Circulate and assist as needed. As well, after the majority of the groups have completed the <u>Hyperbola Cloze the Gap</u> activity sheet, rearrange group members so individuals from different groups can compare and share their results. As a class, address any misconceptions or commonly made errors. *Evaluate (10 minutes)* Once all groups have had a chance to discuss, post the following questions: What information does the standard form of a parabola give us that makes it easier to graph? What information does the standard form of a hyperbola give us that makes it easier to graph?	

How can you tell by examining the coefficients of an equation of the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ where *A* and *B* are not both zero, which conic section the equation represents? What method would we use to rewrite this equation to make it easier to graph? Instruct students to reflect on the activities of the day by answering the following questions: • Did the in-class investigation of parabolas broaden your concept on the multiple forms of parabolas? If so, how so? If not, why not? • Did deriving the standard form of the equation of hyperbola help you better understand the definition of a hyperbola? If so, how so? If not, why not? • What unanswered question do you still have? Post homework assignment & remind students it is due next class. Instruct students to complete exercises related to identifying & sketching various conic sections given their equation and vice versa. A sample set of exercises may be found in the "Student Materials" chapter. *Teacher Reflection* Did this lesson address the needs of more analytical learners? If so, how so? If not, why not? What percentage of the time were students actively engaged in the learning process? How could this lesson be improved?	

Lesson 5 Relating the Many Definitions of Conic Sections (*1-2 class period*)

LESSON SEQUENCE AND DESCRIPTION	TEACHER COMMENTARY
Engage (5 minutes) Post the unit generalizations and concepts. Ask a student to read the unit concepts & generalizations aloud. *Explain (10 minutes)* Tell students they will be working with a group of 3-4 students to discuss and summarize how each of these definitions has been addressed thus far with a focus on one of the conic sections. Also, instruct students to create a summary chart as evidence of their discussions & 2-3 problems (with solutions!) which will be included in the problem bank for the unit test. Remind students they will be presenting their work to the class. Divide students into small groups and assign each small group either circles/ellipses, parabolas, or hyperbolas. *Elaborate (30-45 minutes)* Post expectations for what should be included in the summary chart and what types of problems are appropriate for the unit test. Pass out large pieces of butcher paper for students to summarize their findings. Tell students they have 30-45 minutes for to create their summary charts and sample problems. Circulate and assist as needed. *Evaluate (20-30 minutes)* As each group completes their poster, post the student responses around the classroom. Encourage students to begin writing down the sample problems other groups have created. Ask each group to choose a representative to present the groups summary and sample problems. Encourage students to ask questions of the presenter. When all groups have presented, instruct students to reflect on the activities of the day by answering the following questions in their journals:	Summary charts should include how the assigned conic section is obtained from the intersection of a plane and a right double cone, how the set (or locus) of points satisfies a particular distance condition, and the general equation(s) of the conic section. Neatness, creativity and organization should also be expected. Sample problems should be multi-step constructed response questions (e.g. not multiple-choice) and focused on content consistent with the unit's purpose. NO trick questions! Remind students to write out a solution as well as create a problem.

• With what aspects of your group's summary are you most satisfied? • On what topics could your group (and other groups) provide more explanation? • What unanswered question do you still have? Post homework assignment & remind students it is due the day of the unit review. *Homework* Instruct students to copy down the sample problems each group has designed and create a solution set. Remind students the unit test will be made of these questions. *Teacher Reflection* Did this lesson help students make connections between the multiple definitions of each conic section? If so, how so? If not, why not? What percentage of the time were students actively engaged in the learning process? How could this lesson be improved?	

Lesson 6 Creating with Conic Sections[5] (*1-2 class period*)	
LESSON SEQUENCE AND DESCRIPTION	TEACHER COMMENTARY
Day 1 (Creation) *Engage* Tell students that today's lesson is going to use art to better understand math and math to better understand art. Hand out the <u>Conic Art Project</u> activity sheet[6].	
Explain Review the instructions and answer questions as needed. Instruct students to create a drawing on two-dimensional grid paper using at least one of the standard conic sections (e.g. circle, ellipse, parabola, and hyperbola) and one or more lines. As well, instruct students to list <u>on a separate piece of paper</u> the equations of the conic sections used as well as the domain and range of each equation used. Circulate and assist students as needed.	If examples of student created artwork is available from previous years, an alternate introduction could include a gallery walk and discussion as to what shapes are common to each image. Sample conic artwork can be found in Appendix C.
Day 2 (Communication & Collaboration) *Elaborate* Collect student equation sheets. Redistribute the equation sheets so no student is sitting next to a student with his/her equation sheet. Instruct students to recreate their partners' graphs.	The activity sheets for this lesson may be found in the "Student Materials" chapter.
Evaluate When students have had ample time to recreate the drawings, instruct students to pair up with the individual who had their equation sheet. Encourage students to identify discrepancies and devise methods to improve each student's equation sheet. Continuing with the concept of a minute paper, instruct students to reflect on the activities of the day by answering the following questions: • Did you enjoy this activity? If so, why? If not, why not? • Did creating a drawing using conic sections and lines help you better understand the purpose of each variable in the respective formulas? If so, how so? If not, how could this	Classes on block schedule can complete Day 1 activities as homework to maximize time in class to compare and share with other students.

[5] Source of lesson concept: Leapard, Barbara, B., and Joanne C. Caniglia. "Draw it, Write it, Do it." *Mathematics Teacher* 99.3 (Oct 2005): 152-55. Print.

[6] Source of activity sheet: Villano, Rick. "Conics Art Project." *Mr. Villano's Home Page*. Foothill Technology High School. Web. 11 Feb. 2011. <http://foothilltech.org/rvillano/pdf/algebra2honors/Conics_Art_09.10.pdf>.

activity be adapted to accomplish this goal? • What unanswered question do you still have? Post homework assignment & remind students it is due next class. *Homework* Instruct students to rewrite their equations in calculator form (y =) and recreate their drawings on their graphing calculators using their stated domains. *Teacher Reflection* How did students respond to the opportunity to be creative in math class? Did creating a drawing using conic sections and lines (either original or recreated from a set of equations) help the students understand the purpose of each variable in the respective formulas? What percentage of the time were students actively engaged in the learning process? How could this lesson be improved?	

<table>
<tr><td colspan="2">Lesson 7 Conic Sections in Context Project (2-4 class period)</td></tr>
<tr><th>LESSON SEQUENCE AND DESCRIPTION</th><th>TEACHER COMMENTARY</th></tr>
<tr><td>Engage (5 minutes)
Ask students for examples of conic sections in the real world. Post student responses on the overhead/classroom whiteboard. If student responses are limited, reassure students that by the end of this lesson they will be able to identify multiple applications of conic sections.

Explain (5-10 minutes)
Hand out the <u>Conic Sections in Context</u> hand out. Have a student read the project objectives and expectations aloud. Ask to choose a partner or assign each student a partner.

Tell students they will be using Google Docs to create and present their final project.

Elaborate (60-90 minutes)
In the computer lab or using a mobile computer lab, instruct students to preview one of the tutorials available online, if students are unfamiliar with Google Docs and its applications.

Once students are comfortable with the Google Docs, encourage students to briefly research the topic, outline their presentation, and then research their topic in greater depth before creating their final presentation. Remind students projects will be evaluated based on: accuracy, organization, clarity, and mathematical connections and observations.

Evaluate (45-60 minutes)
Ask each group to present. Students should evaluate each other's presentations using the presentation rubric.

When all groups have presented, instruct students to reflect on the activities of the day by answering the following questions in their journals:

• What was the most surprising and/or instructive discovery you made in class today?
• What unanswered question do you still have?

Post homework assignment & remind students it is due next class.</td><td>As noted, in the solution set to the pretest, applications of conics include: projectile motion, elliptical orbits of the planets and of atoms, ”whispering galleries”, solar cookers, reflecting telescopes, lithotripsy(a medical procedure which uses extracorporeal shock waves to fragment stones in the kidney, bladder or ureter), long range navigation (LORAN) and global positioning systems (GPS)</td></tr>
</table>

Homework Write a 2-3 page paper summarizing various applications of conic sections based on class presentations. Complete the solution set to the class created test questions from the "Relating the Many Definitions of Conic Sections" lesson. *Teacher Reflection* What percentage of the time were students actively engaged in the learning process? How could this lesson be improved? Having completed the instruction portion of the unit with which topics do students still struggle? What steps can you take to address these deficits? What steps do your students need to take to address these deficits?	

Lesson 8	Unit Test Review	(*1 class period*)

LESSON SEQUENCE AND DESCRIPTION	TEACHER COMMENTARY
Engage/Explain (10-15 minutes) Ask students to make groups of 3-4 people. Instruct students to review individual solution sets to the class created test questions in their small group. Circulate and grade students based on completeness. Assign 2-3 problems to each group to present to the class. Circulate and assist as needed. *Elaborate (20-30)* Instruct each group to present the solutions to their 2-3 problems. Encourage students to share alternate methods and to correct (in a different color) their individual solution sets. *Evaluate (5 minutes)* Have students submit their individual solution sets to class created test questions. Post the unit performance objectives. Ask students to take just a few minutes to rate themselves using a scale of: advanced, proficient, nearing proficient, and needs improvement. Post homework assignment & remind students it is due next class. *Homework* Complete Personal Progress Reflection. *Teacher Reflection* Are there discrepancies between your students' self-assessments and your own perception of their abilities? If so, what do you think accounts for the differences? What percentage of the time were students actively engaged in the learning process? How could this lesson be improved?	

Lesson 9	Unit Test	(*1 class period*)

LESSON SEQUENCE AND DESCRIPTION	TEACHER COMMENTARY
Engage (5 minutes) Tell students today is the last day of the “Conic Sections in Context” unit. Thus, today they will be assessed on how much they learned during the past 3-4 weeks. Collect students' Personal Progress Reflections. Instruct students to complete the Unit Evaluation[7]. *Explain (35-40 minutes)* Give students the same test as the <u>Conic Sections Pretest</u>. Working individually, have students complete the post-test. *Elaborate/Evaluate (10-15 minutes)* When all students have completed the pretest, hand back the pretest students took at the beginning of the unit and, as a class, discuss students’ impressions of the unit. Seek constructive criticism and record student observations on the class whiteboard/overhead. At the end of class, thank students for their participation. *Teacher Reflection* As an introductory unit to conic sections, did this unit address the related NM Standards & Benchmarks? If not, what topics still need to be included? Did the students of your class meet 80% or more of the “Conic Sections in Context” performance objectives? With which performance objectives do students continue to struggle? What are the strengths of this alternate unit? What are the weaknesses? Overall, in comparison to a traditional text, which approach do you prefer? Why?	Unit Evaluations may be completed using scantrons or via Survey Monkey.

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[7] Source: "Unit Evaluation Survey for Students." *WIDE World*. Harvard Graduate School of Education. Web. 24 Feb. 2011. <http://learnweb.harvard.edu/wide/courses/files/evaluationsurvey.pdf>.

Student Materials

The list below states the sources of the student materials in the order in which they are introduced within the unit plan. Excluding the "Conic Section Pretest" and the "Conic Sections in Context Webquest", which can found immediately after this page, all the following resources are drawn verbatim from other sources and cited appropriately.

Graphing with Sidewalk Chalk & Rope Activity Sheets

Bush, Ellen R. S. "Illuminations: Human Conics." *Illuminations: Welcome to Illuminations*. National Council for Teachers of Mathematics. Web. 10 Aug. 2010. <http://illuminations.nctm.org/LessonDetail.aspx?id=L815>.

Investigation Ellipses

Cuevas, Gilbert J., Daniel Marks, Ruth M. Casey, Beatrice Moore-Harris, John A. Carter, Roger Day, and Linda M. Hayek. "Algebra Activity - Investigating Ellipses." *Algebra 2*. By Berchie W. Gordon-Holliday. New York: Glencoe/McGraw-Hill, 2005. 432. Print.

Conic Sections

Cuevas, Gilbert J., Daniel Marks, Ruth M. Casey, Beatrice Moore-Harris, John A. Carter, Roger Day, and Linda M. Hayek. "Algebra Activity – Conic Sections." *Algebra 2*. By Berchie W. Gordon-Holliday. New York: Glencoe/McGraw-Hill, 2005. 453-454. Print.

Moiré Pattern Exercises

Hirsch, Christian R., and James Taylor. Fey. "Representing Three-Dimensional Objects." *Core-Plus Mathematics: Contemporary Mathematics in Context*. New York: Glencoe/McGraw-Hill, 2008. 444-45. Print.

Cloze the Gap

Hirsch, Christian R., and James Taylor. Fey. "Representing Three-Dimensional Objects." *Core-Plus Mathematics: Contemporary Mathematics in Context*. New York: Glencoe/McGraw-Hill, 2008. 431-437; 445. Print.

Conic Art Project

Villano, Rick. *Conic Art Project*. *Algebra 2H*. Foothill Technology High School. Web. 22 Dec. 2010.<http://foothilltech.org/rvillano/pdf/algebra2honors/Conics_Art_09.10.pdf>.

Personal Progress Reflection

Benek-Rivera, Joan. "By Teaching You Will Learn: Journals Facilitate Student and Faculty Learning." *MountainRise* 2.1 (2005): 82-91. Web. 22 Feb. 2011. <http://mountainrise.wcu.edu/index.php/MtnRise/article/viewFile/50/82>.

Unit Evaluation Survey

"Unit Evaluation Survey for Students." *WIDE World*. Harvard Graduate School of Education. Web. 24 Feb. 2011. <http://learnweb.harvard.edu/wide/courses/files/evaluationsurvey.pdf>.

Conic Section Pretest

Please answer the following questions to the best of your ability. Use complete sentences and appropriately scaled drawings. Support your responses with well-developed mathematical reasoning. Each question will be graded according to the rubric on page 2.

Conic Sections – a set of curves which are formed when a plane intersects a right double cone

1. List the four conic sections.
2. Describe each of the conic sections in terms of the intersection of a plane and a double cone.
3. Describe each of the conic sections as a locus of points.
4. State the equation of each conic section.
5. Sketch each conic section and label “important” points.
6. Compare and contrast each conic section.
7. Explain some applications of each conic section.

Grading Rubric: New Mexico Rubric for 4-point Open-Ended Items

Score	Description
4	• Offers a correct solution and is well supported by well-developed and accurate explanations. • Gives evidence that an appropriate problem-solving strategy was selected and implemented, but may contain minor errors that do not detract from the overall quality of the student response. • Is clearly organized and focused, and shows a mathematical understanding of the tack or concept. • Contains sufficient work to convey thorough understanding of the problem.
3	• Offers a generally correct solution, but contains minor flaws in reasoning or computation. • Gives evidence that an appropriate problem-solving strategy was selected and implemented, but may contain minor arithmetic or algebraic errors that do detract from the overall quality of the student response. • Is clearly focused, well-organized, but neglects some aspect of the complete solution to the problem. • Lacks significant detail to convey thorough understanding of the task or concept to warrant a complete response.
2	• Offers a partially correct answer to the problem. • May contain flaws indicating an incomplete understanding of the task or concept. • May show faulty reasoning leading to weak answers or conclusions. • May demonstrate unclear communication in writing or diagrams. • May demonstrate a poor understanding of relevant mathematical procedure or concepts.
1	• Offers a correct solution with no supporting evidence or explanation. • Offers little or no supporting detail conveying limited understanding. • Contains numerous errors in computation and reasoning and detracts from the overall quality of the response. • Provides vague interpretation to the solution/explanation, indicating little or no mathematical understanding of the task or concept.
0	• Gives an incorrect response with no work shown. • Offers no mathematical understanding of the problem • Does not address the problem.

(Source: NMPED Assessment and Evaluation Bureau, New Mexico Rubric for 4-point Open-Ended Items, 11 November 2010, http://sde.state.nm.us/AssessmentAccountability/AssessmentEvaluation/dl09/releasedItems/Math%20Gr%204_1_.pdf.)

Conic Section Pretest Solution Set

1. List the four conic sections.
   Answers should include: circle, ellipse, parabola, and hyperbola. Students may also include degenerate conic sections such as a single point, a single straight line, and two intersecting lines.

2. Describe each of the conic sections in terms of the intersection of a plane and a double cone.
   A circle is formed when the intersecting plane is perpendicular to the axis of the cone. An ellipse is formed when the intersecting plane is at an angle less than the slant of the cone. A parabola is formed when the intersecting plane is at an angle equal to the slant of the cone. A hyperbola is formed when the intersecting plane is at an angle greater than the slant of the cone. Note, all of the above exclude the degenerate case.

3. Describe each of the conic sections as a locus of points.
   A circle is a set of points equidistance from a single point. An ellipse is a set of points such that the sum of the distances from a point on the ellipse to two fixed points (e.g. foci) is constant. A parabola is a set of points such that each point is equidistance from a fixed point (e.g. the focus) and a fixed line (e.g. the directrix). A hyperbola is a set of points such that the difference between the distances from a point on the hyperbola to two fixed points (e.g. the foci) is constant.

4. State the standard equation of each conic section.
   Circle: $(x-h)^2+(y-k)^2=r^2$
   Ellipse: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ or $\frac{(y-k)^2}{a^2}+\frac{(x-h)^2}{b^2}=1$ where $a>b$
   Parabola: $(y-k)=a(x-h)^2$ or $(x-h)=a(y-k)^2$
   Hyperbola: $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$ or $\frac{(y-k)^2}{b^2}-\frac{(x-h)^2}{a^2}=1$

5. Sketch each conic section and label "important" points.
   See Appendix A: Sketches of Conic Sections

6. Compare and contrast each conic section.
   Answer may include: A circle is an ellipse where the foci are represented by a single point. A parabola and a hyperbola are both open curves, but parabolas curve away from any asymptote drawn where a hyperbola continually approaches, but never crosses its asymptotes. Each conic section can be defined by a locus of points.

7. Explain some applications of each conic section.
   Answers may include: projectile motion, elliptical orbits of the planets and of atoms, "whispering galleries", solar cookers, reflecting telescopes, lithotripsy (a medical procedure which uses extracorporeal shock waves to fragment stones in the kidney, bladder or ureter[1]), long range navigation (LORAN) and global positioning systems (GPS)

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[1] "Lithotripsy: MedlinePlus Medical Encyclopedia." *National Library of Medicine - National Institutes of Health*. Web. 07 Nov. 2010. <http://www.nlm.nih.gov/medlineplus/ency/article/007113.htm>.

"CONIC SECTIONS IN CONTEXT" WEBQUEST

Introduction

Over 2000 years ago, conic sections became a topic of interest for mathematicians. In 350 BCE, Menaechmus invented the conic sections while trying to solve the Delian problem (e.g. doubling the volume of a given cube). For the next two millennia, mathematicians continued to study the conic sections, yet it wasn't until the 14^{th} and15th century that their usefulness was uncovered. Now, in the year 2011 CE, you are studying the same set of curves. How do conic sections describe and predict natural phenomena? How are conic sections used in architecture, medicine, modern navigation and green technology?

Task

Your task is to answer the infamous question: "When are we ever going to use this?!?" To accomplish this task you are to create a presentation answering one of the above questions. I recommend you briefly research your topic, outline your presentation, and then research your topic in greater depth before creating your final presentation.

Process

Your presentation is to include the following parts:

- An introduction that defines conic sections and lists the four main curves
- Pictures and explanations of at least two real world examples or applications of a conic section
- An explanation of the related conic sections including how each is obtained from the intersection of a plane and a right double cone, how the set (or locus) of points satisfies a particular distance condition, and the general equation(s)
- A conclusion explaining what you learned and what you found interesting

All presentations are to be submitted on **3/18** for 3A and **3/21** for 2B, 3B. It is due **at the start of the class period even if you are absent**. Late projects will not be accepted.

Resources

The following web pages are a good place to start. You are encouraged to search for others.

http://www.britannica.com/EBchecked/topic/132684/conic-section/235552/Post-Greek-applications
http://britton.disted.camosun.bc.ca/jbconics.htm
http://mathcentral.uregina.ca/qq/database/QQ.09.02/william1.html
http://jwilson.coe.uga.edu/emt669/Student.Folders/Jones.June/conics/conics.html
http://www.ccathsu.com/files/handouts/Parabolic%20Solar%20Cookers.pdf

Evaluation

The following is the grading rubric for your project.

Criteria	Beginning	Developing	Accomplished	Exemplary
Visual Appeal (8 points)	Use of font, color, graphics, effects etc. that detract from the presentation content	Use of font, color, graphics, effects etc. which somewhat enhance the presentation of content	Use of font, color, graphics, effects etc. which enhance the presentation of content	Use of font, color, graphics, effects etc. which exceptionally enhance the presentation of content
Organization of Content (8 points)	No clear or logical organizational structure	Content is logically organized for the most part. Some content is irrelevant or inappropriate.	Content is logically organized. Most content relevant and appropriate.	Content is logically organized. All content is relevant and appropriate.
Subject Knowledge (16 points)	No mathematical reasoning or justification for reasoning is present.	Some mathematical reasoning or justification for reasoning is present.	Mathematical reasoning and justification for reasoning is present.	Exceptional mathematical reasoning and justification for reasoning is present.
Mechanics (8 points)	More than 5 grammatical and/or spelling errors	5 grammatical and/or spelling errors	3-4 grammatical and/or spelling errors	1-2 grammatical and/or spelling errors
Presentation (16 points)	Delivery not smooth and audience attention often lost	Delivery not smooth, but able to maintain interest of the audience most of the time.	Rehearsed with fairly smooth delivery that holds audience attention most of the time.	Well-rehearsed with smooth delivery that holds audience attention.
Content (40 points)	More than one requirement was not completely met.	One requirement was not completely met.	All requirements are met.	All requirements are met and exceeded.
Citation (4 points)	Does not cite information correctly	Does not consistently cite information	Cites information correctly and consistently	Cites information correctly and consistently when referring to specific images and content

Total: ____/100

Peer-Evaluations

On a separate piece of paper, recreate the below table to grade your classmates' presentations.

Fill in the student's initials and topic.
Use the following symbols to grade each element:

+=Yes 0=Somewhat - =No

Student's Name:		
Student's Topic:		
1. Mathematical representations are appropriate and accurate		
2. Mathematical connections and observations are explained		
3. Vocabulary is appropriate to the content and the audience.		
4. Font, color, graphics, effects, etc. support the presentation and are visually appealing.		
5. The presentation is well-organized and content is relevant.		
6. Delivery is well-rehearsed and holds the audience's attention.		
7. Speaker is well-dressed.		
8. Speaker engages audience members.		
9. Speaker gives audience time to think.		
10. Speaker responds well to questions.		

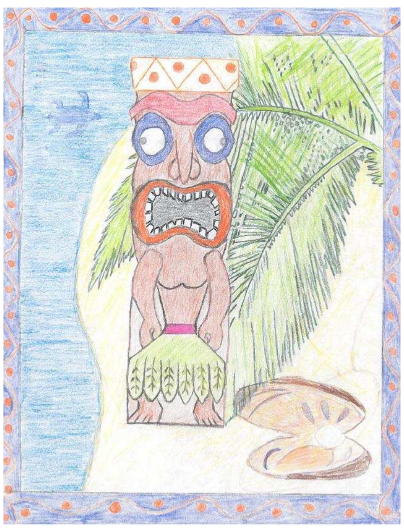
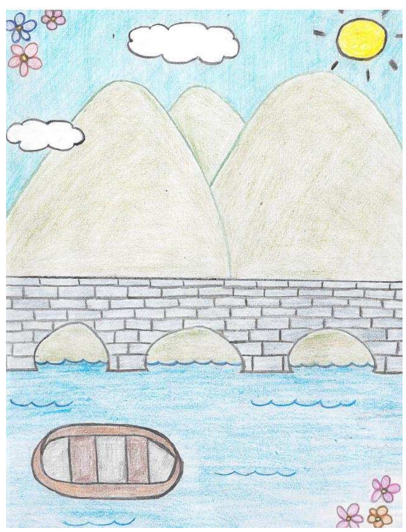
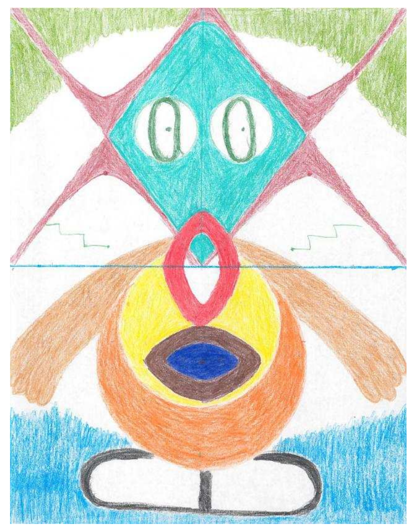
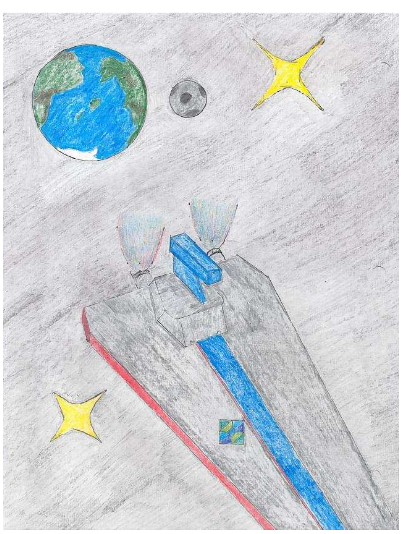
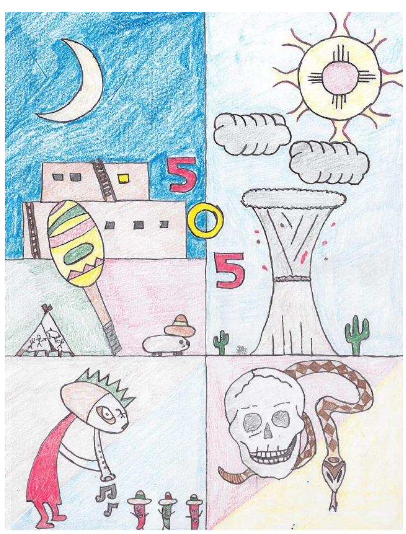
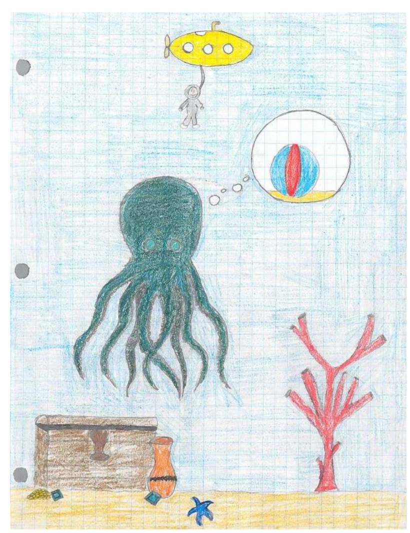
Results

Implementation of this unit occurred March of 2011 at Los Alamos High School. This chapter analyzes the work of the students and staff involved by reviewing samples of students work, comparing students' pretest and post-test results, summarizing the reflections of students and colleagues, and evaluating the personal progress and unit evaluation surveys completed by the students involved.

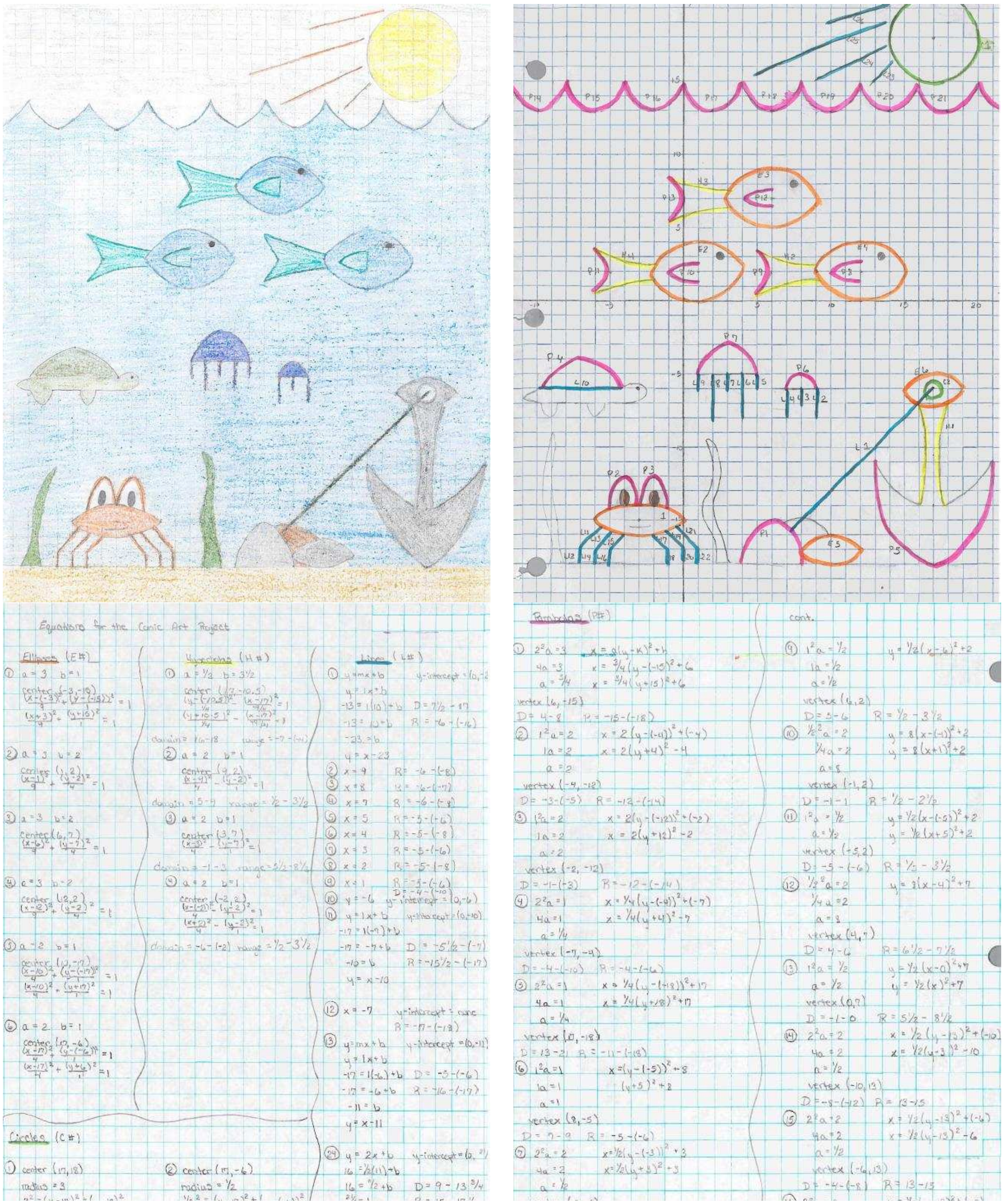
Samples of Student Work

Student work in this unit included in-class activity sheets, etymology poster board displays, and conic section summary charts. Each of these tasks culminated in two projects: the conic art project and a conic section webquest.

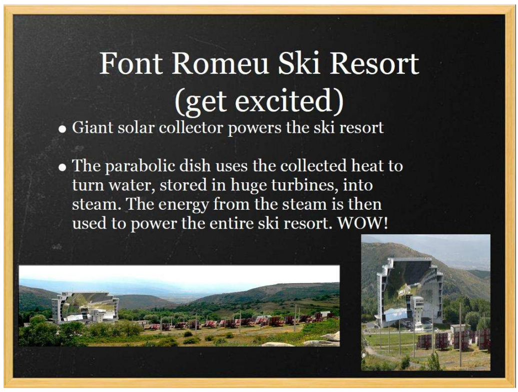
As can be seen for the selection of images below, students displayed an amazing amount of creativity in the construction of their conic art.

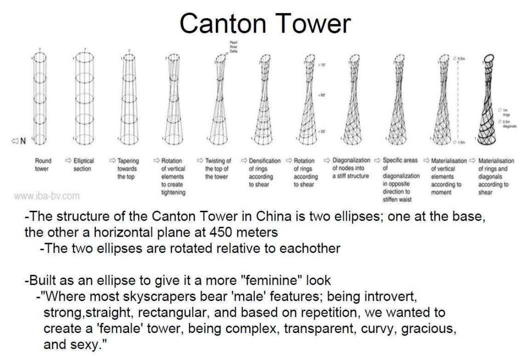







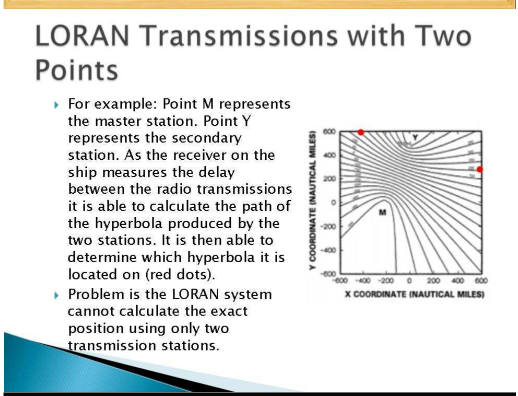
Equally astounding was the time and effort students put into deducing the equations for the 4 ellipses, 2 hyperbolas, 2 circles, 10 lines, and 2 parabolas that were the skeleton of their creations. One example of this endeavor can be seen in the below student-generated conic art sample and its related equation sheets.

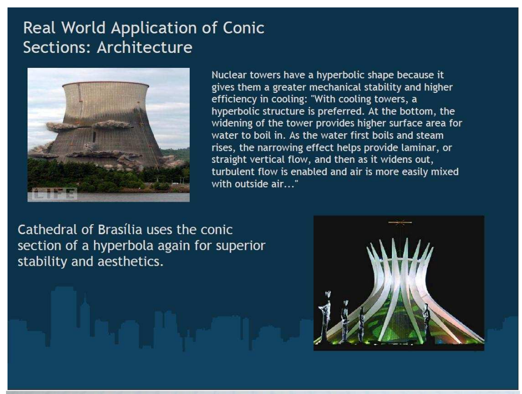


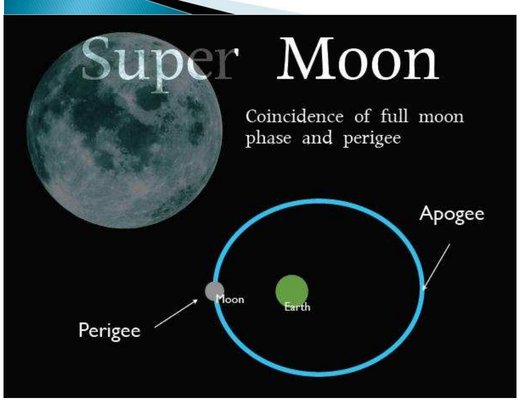
In addition to the conic art project, students researched and created a presentation on one of two topics: how conic sections describe and predict natural phenomena or the applications of conic sections in architecture, medicine, modern navigation and green technology. As can be seen in the screen shots below, students did an exceptional job of summarizing & presenting on a multitude of topics including: projectile motion, elliptical orbits of the planets and of atoms, "whispering galleries", solar cookers, nuclear cooling towers, reflecting telescopes, lithotripsy, long range navigation (LORAN) and global positioning systems (GPS).

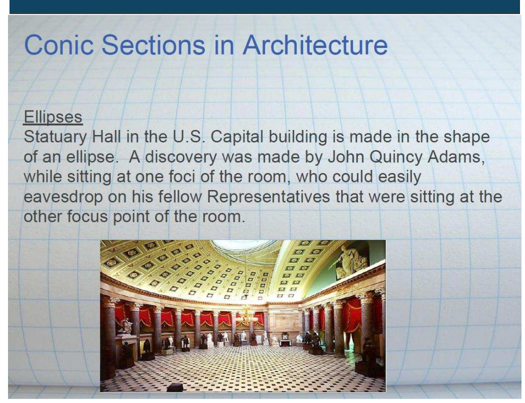












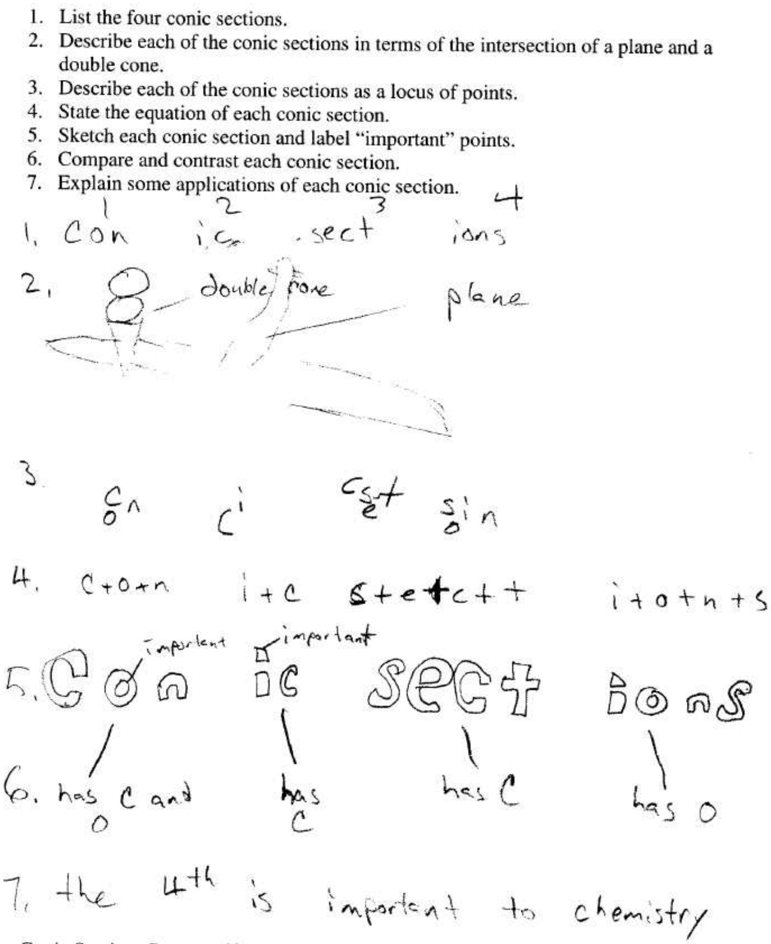
All in all, these two projects allowed students to adapt the assignments to their skills and interests, encouraging students to invest in the unit personally as well as academically. A comparison of pre- and post-test results indicates significant improvement.

Comparison of Students' Pretest & Post-test Results

As noted in the Conic Sections in Context unit plan, students completed a pretest prior to beginning the unit. As can be seen in the table below, questions addressed each of Bloom's Taxonomy of Learning.

Category	Example
Knowledge	List the four conic sections. State the equation of each conic section.
Comprehension Application	Describe each of the conic sections in terms of the intersection of a plane and a double cone. Describe each of the conic sections as a locus of points.
Analysis Synthesis	Sketch each conic section and label "important" points. Compare and contrast each conic section.
Evaluation	Explain some applications of each conic section.

Of the ninety-one students who completed the pre-test, three scored above 0 out of 28. Of these three students, two had transferred into Algebra 2 from Algebra 2/Trig at the beginning of second semester and had already studied conic sections. The other student is a freshman who had studied conic sections in his enrichment class for gifted students. Of the three students, the two Algebra 2/Trig transfer students scored 9 out of 28 (~32%) and 6 out of 28 (~21%); the other student scored a 10 out of 28 (~36%). The remaining 88 students of the class responded with "I don't know", left the questions blank, or used the space to doodle as can be seen in the student-generated example below.


1. List the four conic sections.
2. Describe each of the conic sections in terms of the intersection of a plane and a double cone.
3. Describe each of the conic sections as a locus of points.
4. State the equation of each conic section.
5. Sketch each conic section and label "important" points.
6. Compare and contrast each conic section.
7. Explain some applications of each conic section.

1. Con ic sect ions
2. double cone plane
6. has C and O has C has C has O
7. the 4th is important to chemistry

Following the completion of the Conic Sections in Context unit, student scores on the post-test averaged 23.125 out of 28 (~83%) with a median of 23.25 out of 28. For a point of comparison, students from my Algebra 2 class last year averaged a 70.05% on the summative evaluation of conic sections.

Summary of Student and Staff Reflections

Though an improvement in student scores is certainly desired, the greatest change I celebrated was in the mentality of my students. Throughout the Conic Sections in Context unit, students were asked to summarize and reflect on what they had learned during the unit thus far. At the conclusion of the unit, each student synthesized his/her reflections into a single paper, the personal progress reflection. As noted in the preface, the goal of these reflections is three-fold: improve self-awareness, learn by analyzing experiences, and reinforce understanding by retention of concepts[1].

These reflections provided an invaluable forum for students to share their thoughts and feelings on the unit overall. I was delighted by the overwhelming positive reception of the unit by my students. As one student commented, “I felt that I learned more by working with conic sections rather than using the book.” Similarly, another student remarked, “Personally, I thought this unit would be a normal boring math section. I imagined countless equations and exercises and pointless attempts at explaining how this is going to affect you. However, this was not true. This unit expected you to think and explore.” Likewise, another student reiterated this sentiment when she stated: “It was a truly awesome experience to actually construct math so to speak… Math has never been my strong point, but during this section I really did feel confident. It gave me a feeling of accomplishment when I understood each concept.” Less than a handful of my students, however, found the approach of this unit taxing and noted a preference for direct instruction. As stated by one student, “I don't think the method of experimenting on our own at the beginning of the lesson was beneficial to me. Also, the activities outside were confusing and I don't think I learned much from them.” Nonetheless, this same student concludes her reflection by observing, “But I thought changing our routine was nice and gave a little variety.” Overall, my students relished the opportunity to learn math in a new way. As was eloquently stated by one student, “this unit not only taught me the material more efficiently, but it also taught me more about myself. I realize now that I can do well in math and enjoy it, as well as how I learn best. These different varieties of teaching tactics had a major impact on my understanding.”

Though my students expressed positive sentiments towards this unit overall, different instructors experienced varying levels of success with this unit. Within Los Alamos High School (LAHS), there are three instructors who teach Algebra 2- myself, Christopher Cretella, and John Pawlak. John has taught Algebra 2 for three years at LAHS where I have taught it for two years and Christopher for one. The advice and insight of these two gentlemen has been invaluable to the implementation and evaluation of this unit. Though neither implemented the Conic Sections in Context unit in its entirety, both gentlemen used multiple lesson plans from the unit within their classrooms. John found that his stronger students did not like the lessons where his weaker students enjoyed it, but didn’t show any significant improvement in their overall proficiency compared to his students from last year[2]. Christopher found the design effective in that it

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[1] Benek-Rivera, Joan. "By Teaching You Will Learn: Journals Facilitate Student and Faculty Learning." *MountainRise* 2.1 (2005): 82-91. Web. 22 Feb. 2011. <http://mountainrise.wcu.edu/index.php/MtnRise/article/viewFile/50/82>.

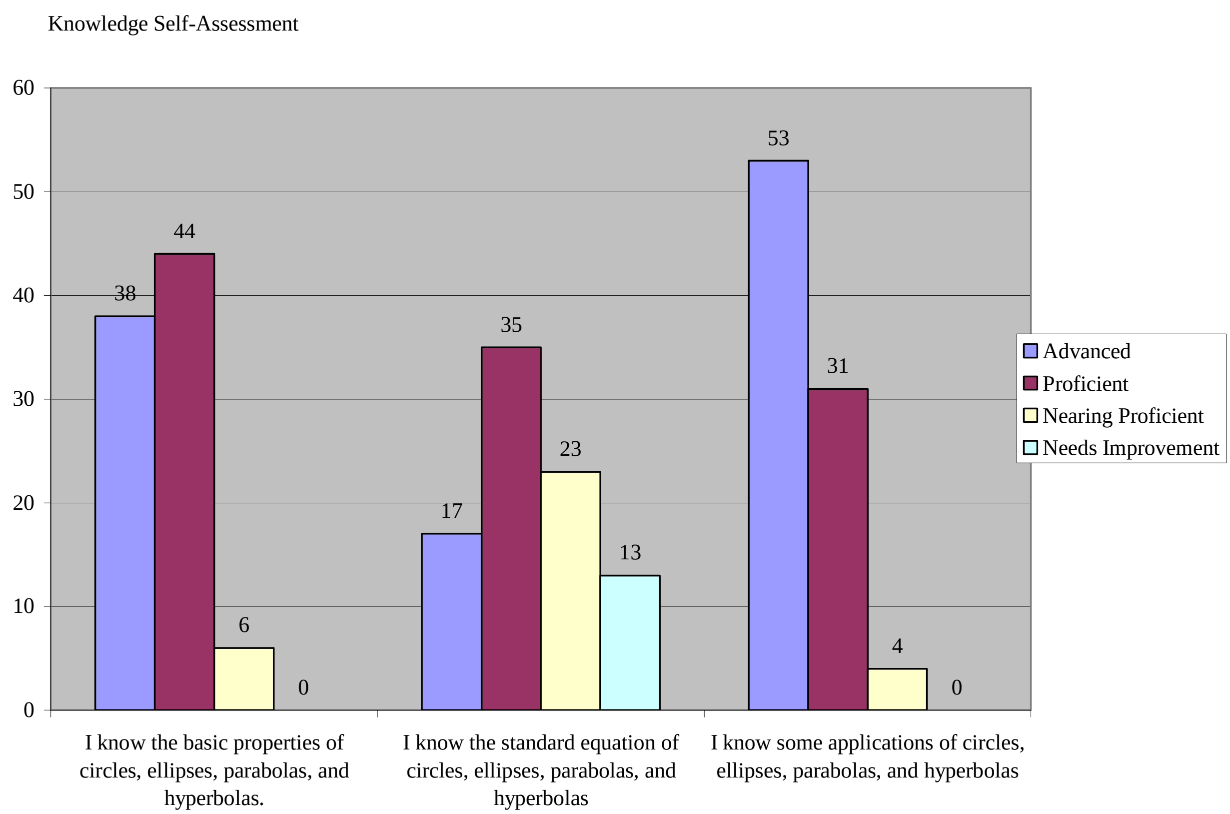
[2] Pawlak, John. Letter to Elizabeth Richardson. Mar. 2011. Instructor Reflections. Los Alamos High School. Los Alamos.

“covered each aspect of conic sections in multiple ways”, but struggled with the implementation due to time constraints and “students’ reluctance to learn through discovery.”[3] It is difficult to determine why this unit was more successful with some instructors than with others, but teaching style and personality are certainly influencing factors. As noted by John after attempting the etymology lesson with his class and having the lesson fail, “I think it's as much my problem as theirs. We're just not used to doing something like that in a math class.” Thus, I would argue that this unit is not designed to be universally implemented, but if it suits the teaching style of the instructor and the learning style of the students, it can be quite successful.

Evaluation of the Personal Progress and Unit Evaluation Surveys

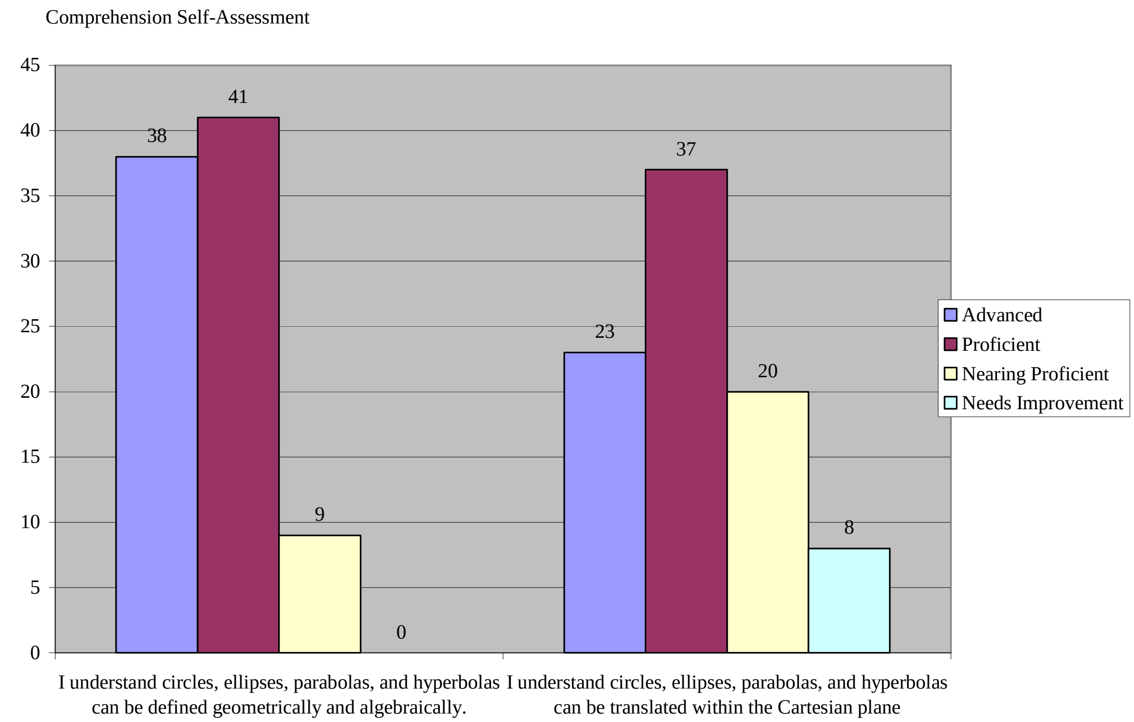
In addition to personal progress reflections, students completed two surveys at the conclusion of the Conic Sections in Context unit. Each student completed a personal progress survey to evaluate his/her success with respect to the unit performance objectives and a unit evaluation survey to assess the effectiveness of the unit.

The series of charts below summarizes student responses to the self-assessment survey. Note: of the ninety-one students who participated in the unit, eighty-eight responded. Of the eighty-eight who responded, students were more confident of their understanding of the applications of conic sections than their knowledge of the standard equations.

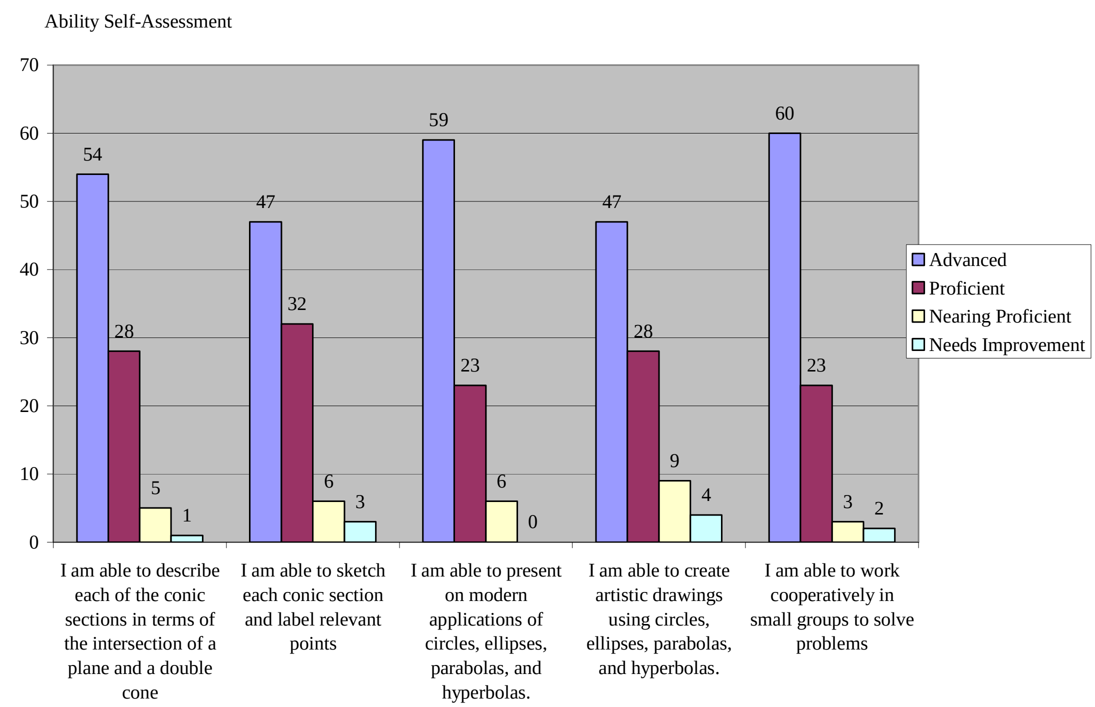


[3] Cretella, Christopher. Letter to Elizabeth Richardson. Mar 2011. Instructor Reflections. Los Alamos High School. Los Alamos.

Likewise, students were more confident of understanding the connection between the geometric and algebraic definitions of conic sections than the translation of the conic sections within a Cartesian plane.

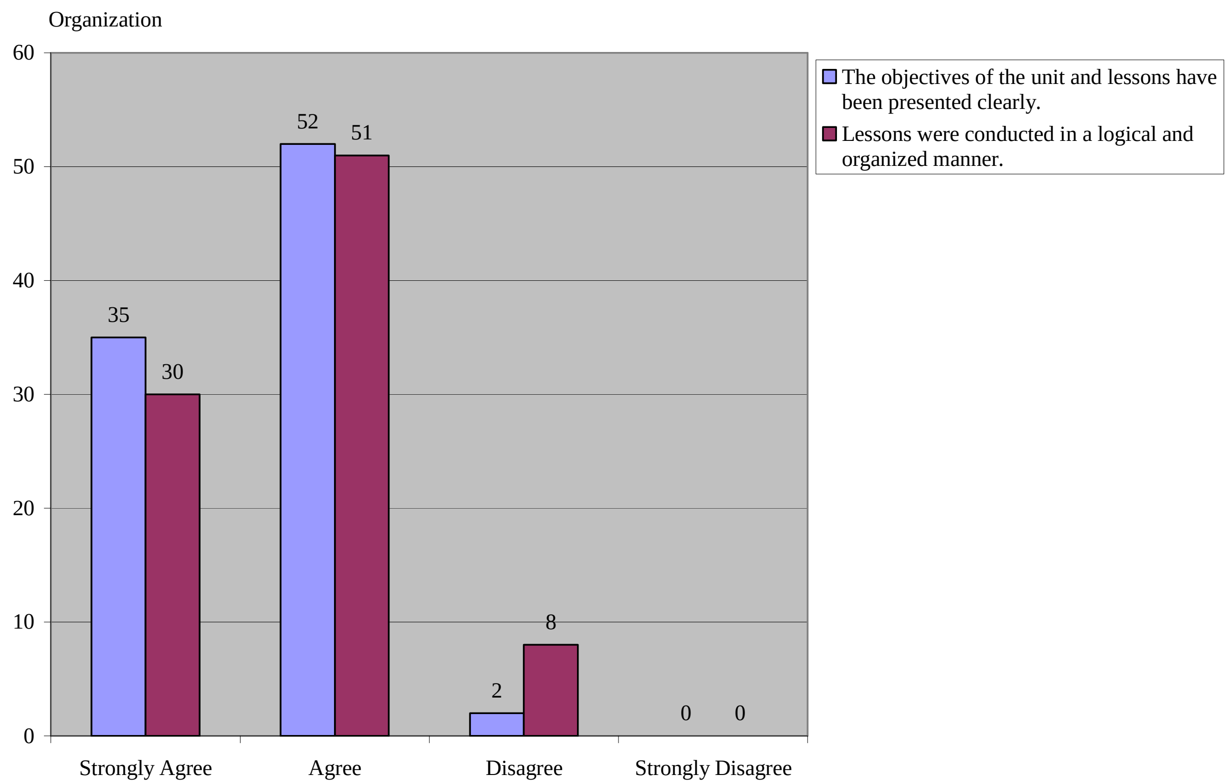


Lastly, students showed little hesitation in their abilities to accomplish the tasks related to the Conic Sections in Context unit.

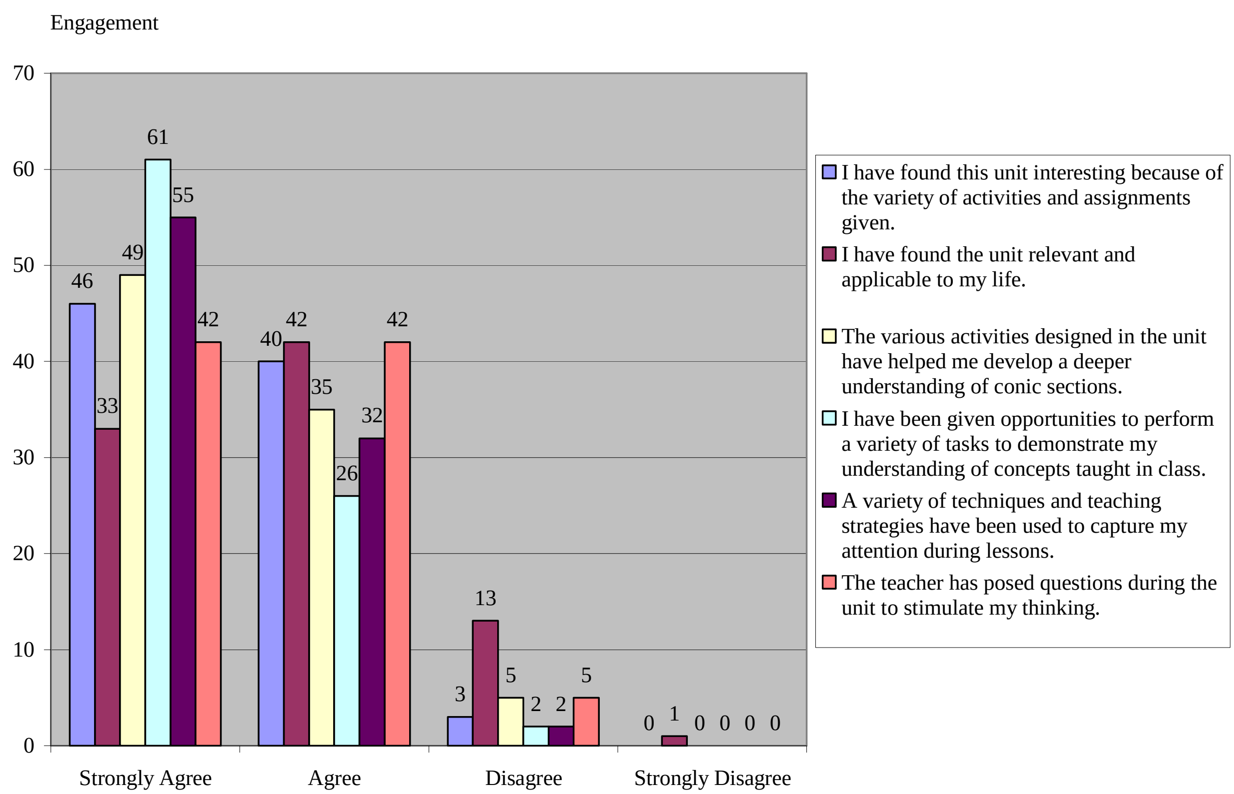


Interestingly, students' performance on the Conic Sections post-test matched their personal assessment of their abilities.

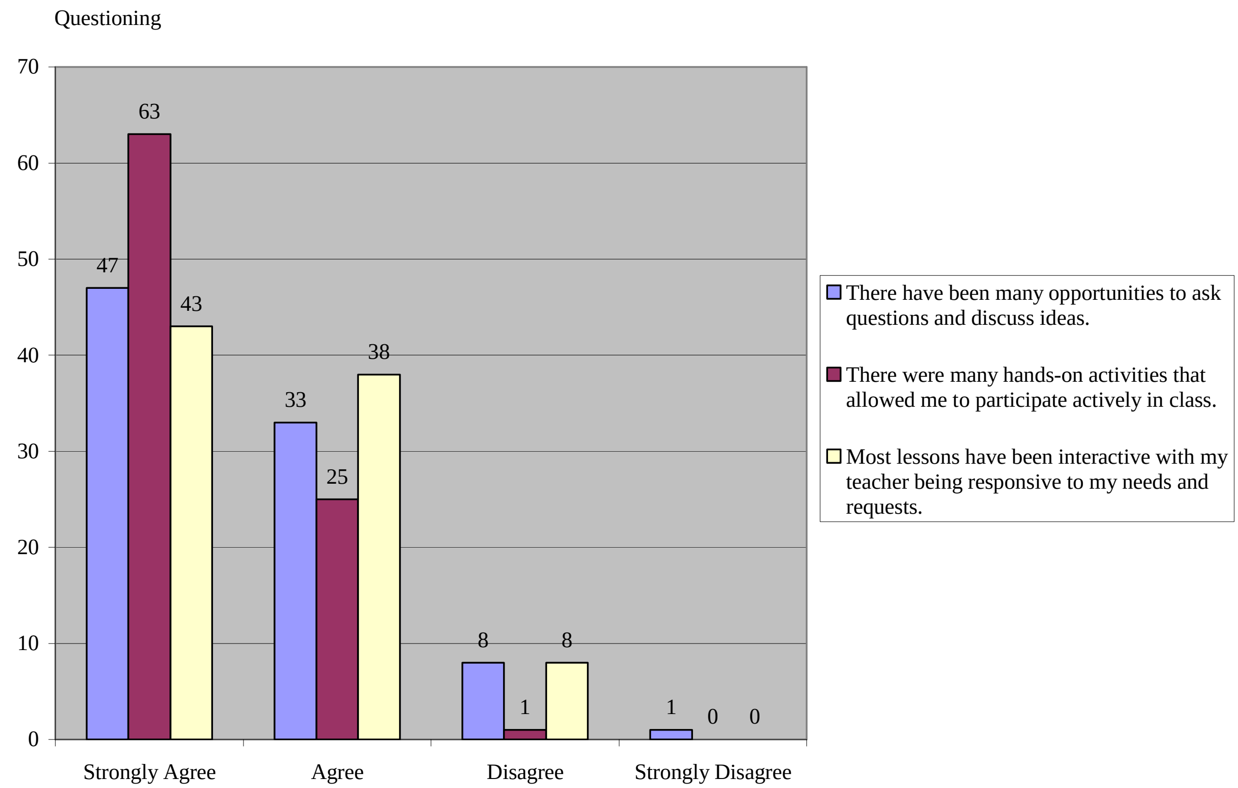
In regards to Unit Evaluation survey, as with the student reflections, student responses to the Conic Sections in Context unit were overwhelming positive for the most part. Of the ninety-one students who completed the unit, eighty-nine completed the unit evaluation survey. In regards to organization, the vast majority of the students felt that the objectives of the unit and lessons were presented clearly and the lessons were conducted in a logical and organized manner as can be seen in the chart below.



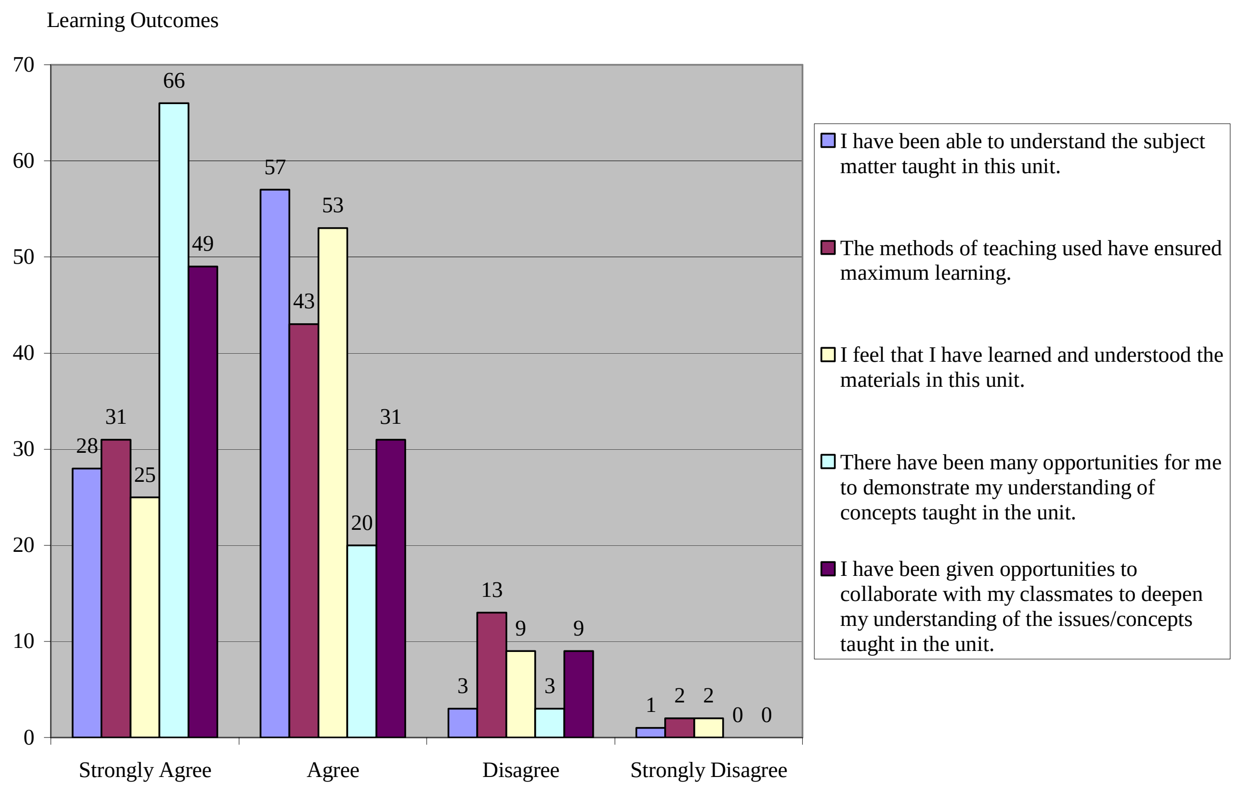
Likewise, as is seen in the engagement summary chart below, though a little over a dozen students felt the unit was not relevant and applicable to their lives, over 96% of the students found the unit interesting because of the variety of activities and assignments given. Similarly, 98% of students felt a variety of techniques and teaching strategies had been used to capture his/her attention during the lessons. Equally, the same percentage of students felt he/she had been given opportunities to perform a variety of tasks to demonstrate his/her understanding of concepts taught in class.



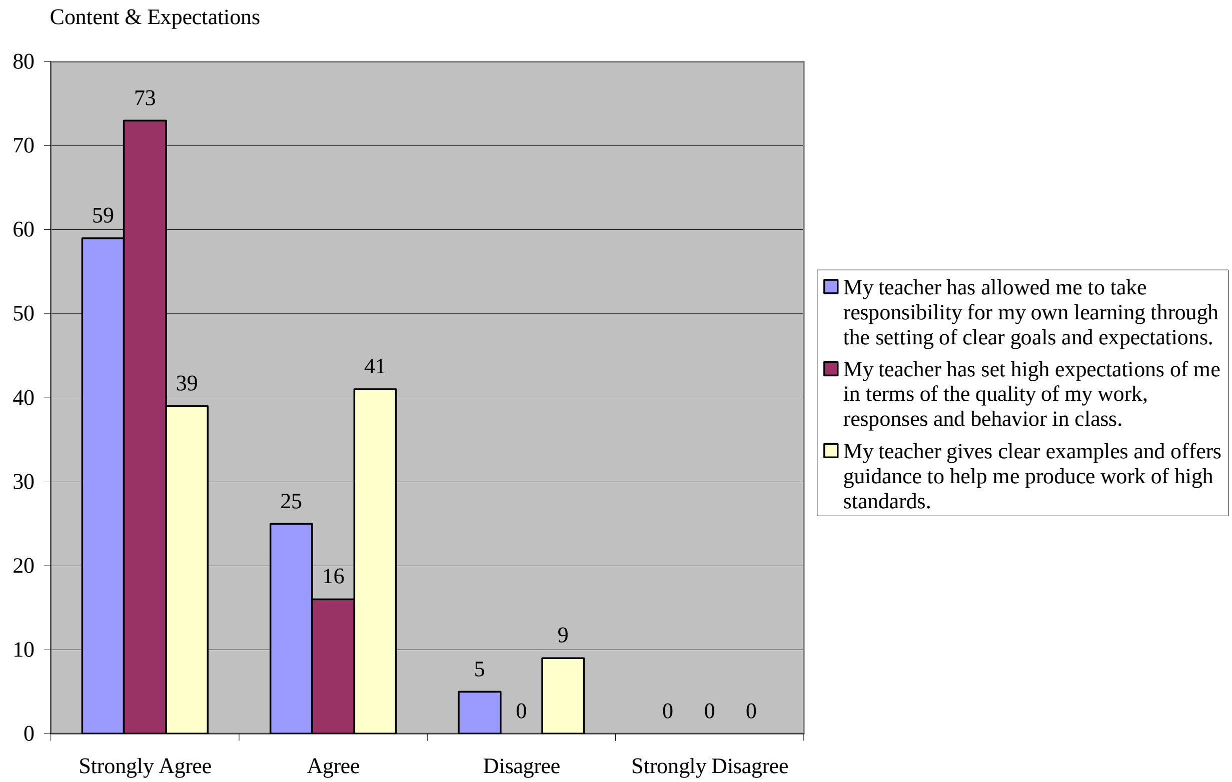
Conversely, as is seen in the questioning summary chart below, 9% of students felt that there had not been many opportunities to ask questions and discuss ideas – a shortcoming perhaps due to student absences or the time constraints of the unit. Consequently, possible solutions to this issue are discussed in the conclusion.

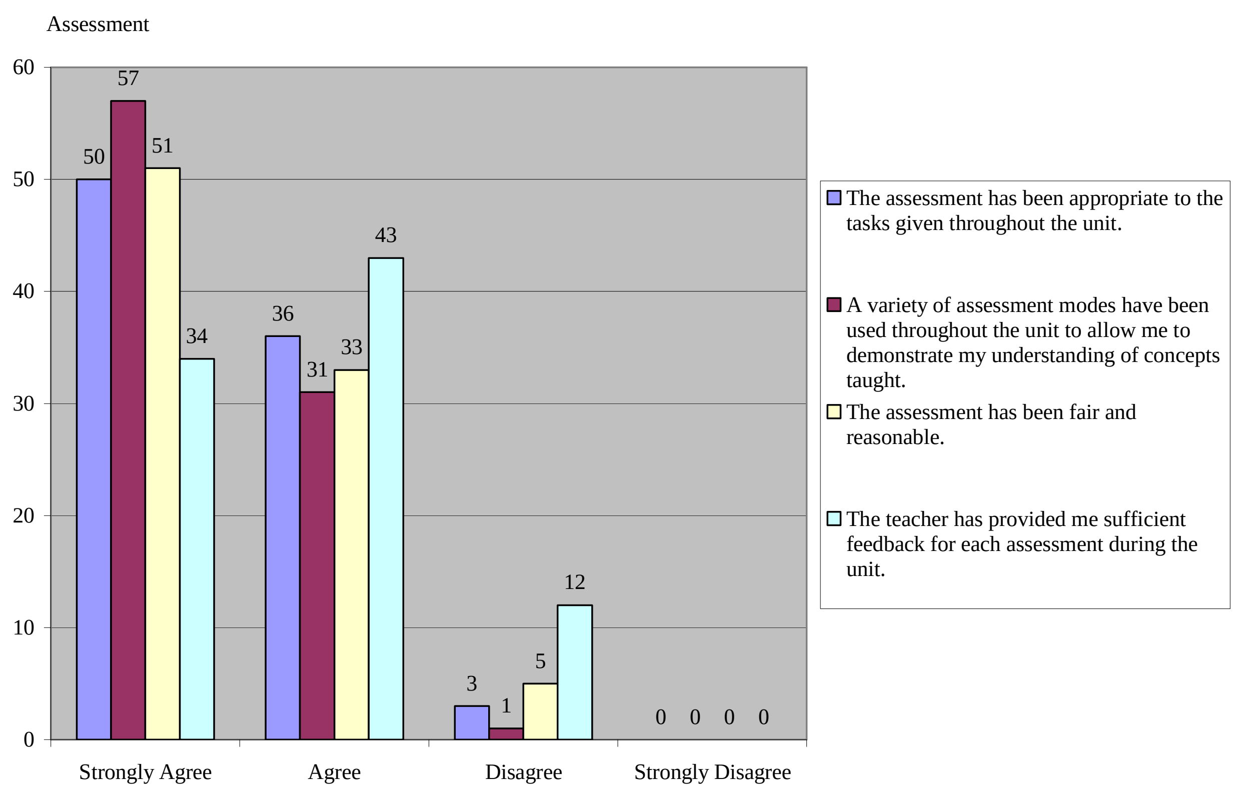


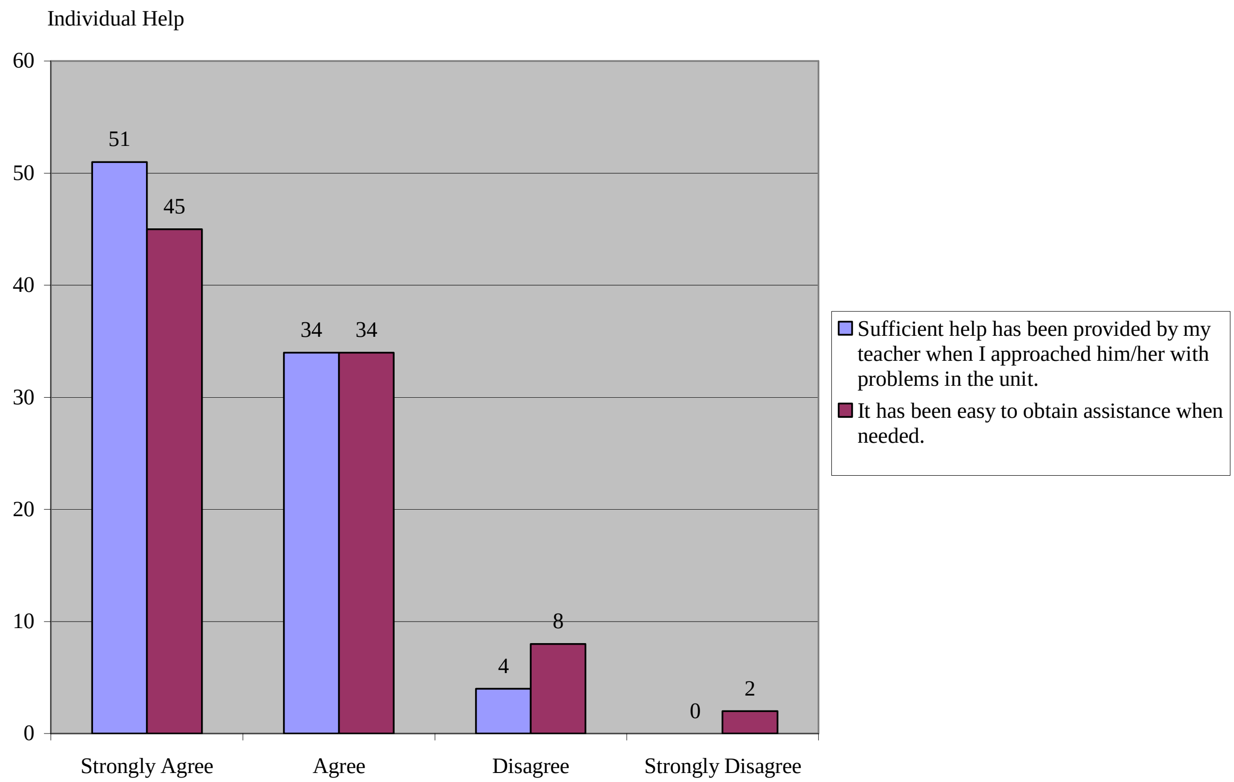
Likewise, as is seen in the below learning outcomes summary chart, though 96% of students felt that he/she understood the subject matter taught in this unit, only 83% of students felt that the methods of teaching used ensured maximum learning. Thus, as will also be discussed in detail in the conclusion, there is still room for improvement.



Lastly, as is seen below in the content and expectations, assessment, and individual help summary charts, instructor implementation style certainly influences the success of the unit. As noted, with the content and expectations summary chart, 100% of students felt that I set high expectations of them in terms of the quality of their work, responses and behavior in class. However, as identified throughout the three charts, roughly 10% of students felt I could improve by offering clear examples and additional guidance in the form of sufficient feedback and greater availability for students to obtain assistance.







Thus, data supports the conclusion that this unit successfully fulfilled the target goals and that opportunity does exist for improvement in its design as will be discussed in detail in the next chapter.

Conclusion

This chapter provides an overall assessment of the Conic Sections in Context project in regards to what worked, what didn't work, and how I plan to change the project when I implement it again in the future. The chapter concludes with a brief reflection on the goal of this independent study.

What Worked

The success of the Conic Sections in Context unit within my own classroom was largely due to my students' willingness to experiment with the concepts presented. Knowing I would implement this unit in the second semester, I actively worked to build trust among my students. Thus, when intellectual risks were presented, students were confident that they and their classmates could succeed. The expectation of success began with my students taking the pre-test. Though some students were initially intimidated by the content of the pre-test, all were reassured once they understood that the post-test addressed the same topics. The use of a pre- and post-test was invaluable in establishing the performance objectives of the unit at the outset of the unit.

Likewise, as the first lesson, the "Discovering Conic Sections through Technology" lesson engaged students' interest by using a familiar, age-appropriate tool— the computer. As noted by my colleague, John Pawlak, students "were able to use the software to visualize the essential concepts."[1] Similarly, students truly enjoyed working outside during the "Graphing with Sidewalk Chalk & Rope" lesson. Though this lesson demanded an exceptional amount of classroom management, the energy and excitement experienced by the students was well worth it. In particular, kinesthetic learners, whose learning style is rarely integrated into a traditional math classroom, were ecstatic for the opportunity to use their bodies to better understand the concepts. As an immediate follow-up to this lesson, the use of Moiré Pattern homework helped emphasize and clarify the concept of the distance-definitions. In the same manner, the "Relating the Many Definitions of Conic Sections" lesson gave students the opportunity to pause, reflect with their peers, and regroup so as to better understand the key concepts of the unit. As one student commented in her personal progress reflection, "I got very frustrated when trying to figure out all the equations for my summary chart, but ended up loving the fact that I had made that, for it helped me a lot with the future activities."

The final series of projects— the conic art project and personal progress reflections— were also indispensable assessment tools of student success and unit effectiveness. The conic art project gave students the opportunity to express themselves artistically while also challenging them to find the related equations. The variety of student work also helped clarify deficiencies in the unit such as defining the domain and range for a given conic section. Likewise, the personal progress reflections created a forum for students to summarize the activities of the unit and discuss their

[1] Pawlak, John. Letter to Elizabeth Richardson. Mar. 2011. Instructor Reflections. Los Alamos High School. Los Alamos.

thoughts and feelings. In reviewing the reflections of my students, I was truly impressed with their metacognitive skills. Student comments were thoughtful and well-stated. Most importantly, the reflections gave students the opportunity to offer constructive criticism and shape how the unit would be conducted in the future. In the end, the trust I had worked to build among my students prior to this unit ensured a successful conclusion of this unit.

What Didn't Work

The counterpoint to these successes were the few aspects of this unit that didn't work including one lesson and three logistical issues.

Of the ten or so lessons of this unit, the "Clozing the Gap" lesson was a true disaster. After multiple interactive lessons which involved computers or going outside, students were "caught off guard with having to explore a more rigorous approach to learning conic sections"[2] as my colleague, Christopher Cretella noted. Students were frustrated and angry throughout the lesson which made it difficult, if not impossible, for learning to actually occur. In the end, some students came in for help after school while other students received helped from family & friends or turned to the internet for online tutorials. Sadly, a handful of students gave up which was evident when grading their conic art projects.

In addition to the "Clozing the Gap" lesson, the three logistical issues which created situations that didn't work were: time constraints, absences, and grading. In regards to time, when planning the unit, I expected to have the month of March to implement the unit. In reality, a staff in-service occurred March 11th and Standards Based Assessments were conducted March 22nd-24th. The loss of these four instructional days severely impacted the implementation of the unit as well as student stress levels. As well, March is peak allergy season and a busy time for school activities and athletics. Thus, many students missed key lessons requiring me to create alternative assignments. Compounding the time involved to create alternative assignments, the volume of grading involved in this unit was at times overwhelming. In particular, the conic art project required an immense investment of time to grade as each student submitted 22 unique equations. By encouraging creativity and originality, I eliminated the redundancy which usually exists when grading a traditional test which increased my time grading exponentially. Nonetheless, I strongly believe the conic art project was well worth the effort of my students and me.

How I Plan to Change the Project

Overall, I am eager to implement this unit in my Algebra 2 classes next year. In particular, I am looking forward to addressing the same content, but reorganizing the general structure of the unit. First and foremost, I plan to have the students complete the webquest at the beginning of the unit as a complement to the "Discovering Conic Sections through Technology" lesson. I originally placed this lesson at the end of the unit believing students would need a strong

[2] Cretella, Christopher. Letter to Elizabeth Richardson. Mar. 2011. Instructor Reflections. Los Alamos High School. Los Alamos.

background in the properties of conic sections to fully appreciate the abundant applications of conic sections. However, multiple students stated in their personal progress reflections that the webquest gave purpose to the unit and, as one student stated, helped “connect practical applications of the conic sections from the classroom to real life.” By including the webquest within the first lesson of the unit, the depth of mathematics involved could not be addressed, but the student-generated examples could be continuously referred to throughout the remainder of the unit.

As well, the variety of approaches within this unit met the needs of a multitude of learning styles, but each lesson did not. In fact, each lesson targeted a specific learning style. To address the needs of all students in each lesson, next year I plan to focus on one conic section each class using a variety of the approaches. For example, after the “Discovering Conic Sections through Technology” lesson and the webquest, students will learn about circles by graphing circles using sidewalk chalk and rope, deriving the equation in a small group, and taking notes based on a short in-class lecture. Likewise, each of the next 3 lessons would involve the same activities but attend to a different conic section each day. My hope is that by repeating the variety of activities during each lesson the needs of all students will be addressed and students will remain engaged while tackling some rigorous mathematics.

Lastly, in addition to reorganizing the unit, I plan to restructure my assessment of student success by providing options from which students can choose. This year, as noted in the unit plan, student assessment included the conic art project, the webquest, the student-generated test questions, and the post-test. As noted in multiple student reflections, this abundance of assessment resulted in students being frustrated and overwhelmed. Next year, I plan to differentiate and allow students to choose to construct a test, do an art project, or take a test. My hope in providing these options is for students to be highly invested in their assessment choice and thus strive for excellence.

Goal of this Independent Study

In conclusion, after a year of working on creating, implementing, and now evaluating this independent study, I am confident that I met the goal of my project. As stated in the introduction, this project is a discovery-based, multi-sensory unit composed of a series of lessons designed to teach high school students about conic sections. The overall goal is for students to be actively engaged in the process of discovery, reflection, and creation. From my own classroom observations and the results presented in the last chapter, I have no doubt that this goal was met and am delighted to have it become a permanent component of the Algebra 2 curriculum of Los Alamos High School.

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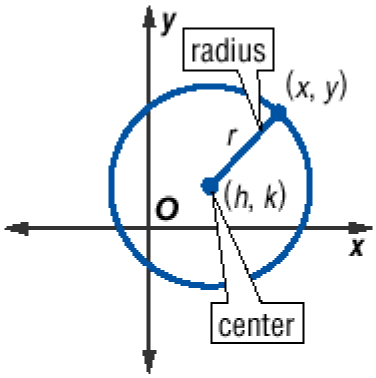
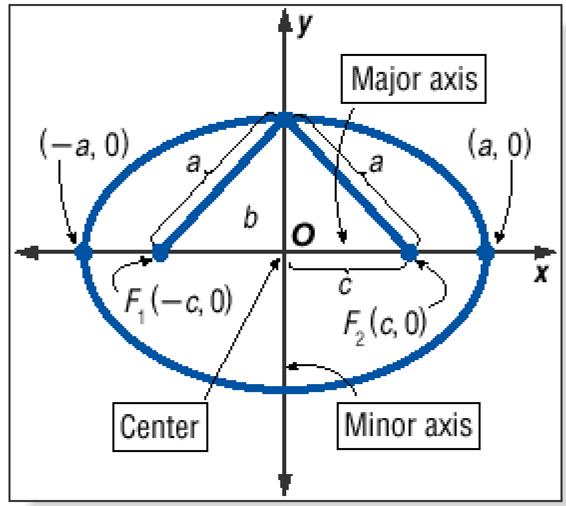
APPENDIX A: CONIC SECTIONS in PLANAR VIEW

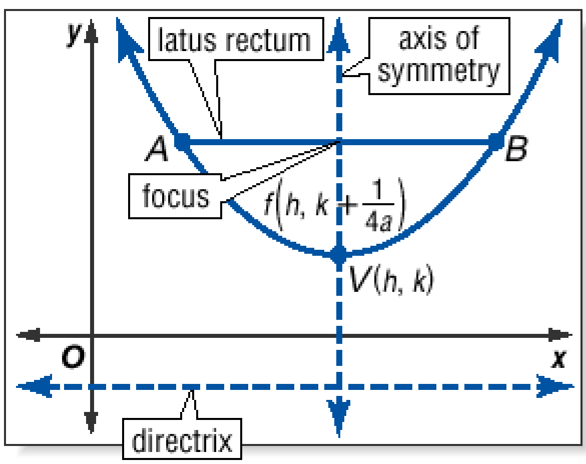
CIRCLE



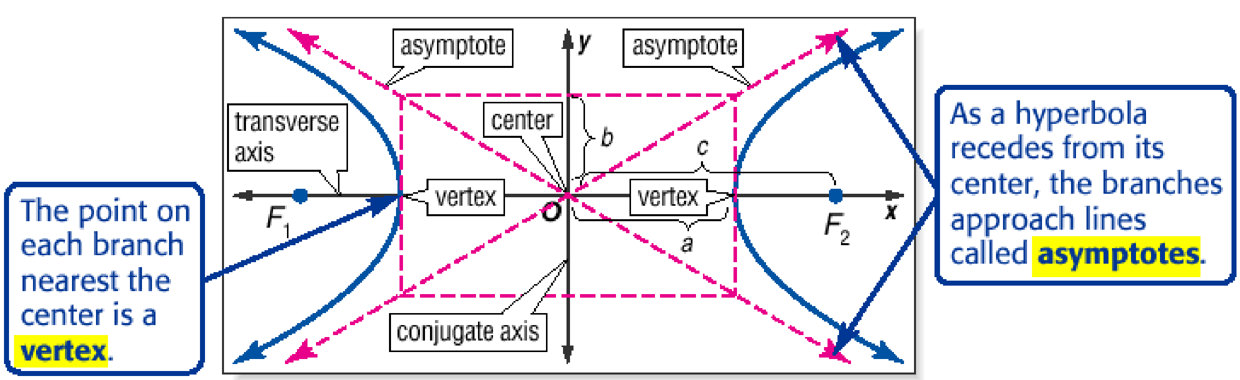
ELLIPSE



PARABOLA

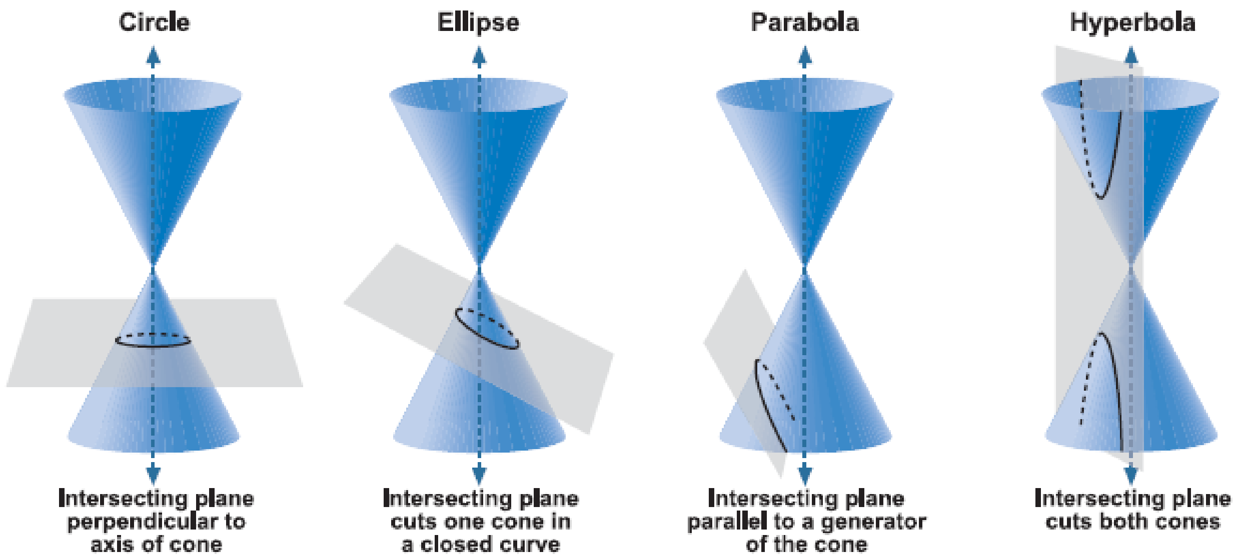


HYPERBOLA



Source: Cuevas, Gilbert J., Daniel Marks, Ruth M. Casey, Beatrice Moore-Harris, John A. Carter, Roger Day, and Linda M. Hayek. "Algebra Activity - Investigating Ellipses." *Algebra 2*. By Berchie W. Gordon-Holliday. New York: Glencoe/McGraw-Hill, 2005. 421; 426; 434; 442. Print.

APPENDIX B: CONIC SECTIONS in 3D



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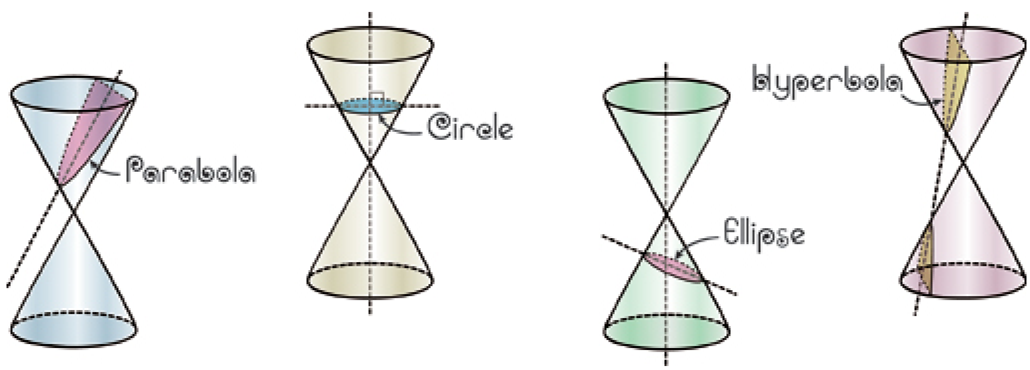
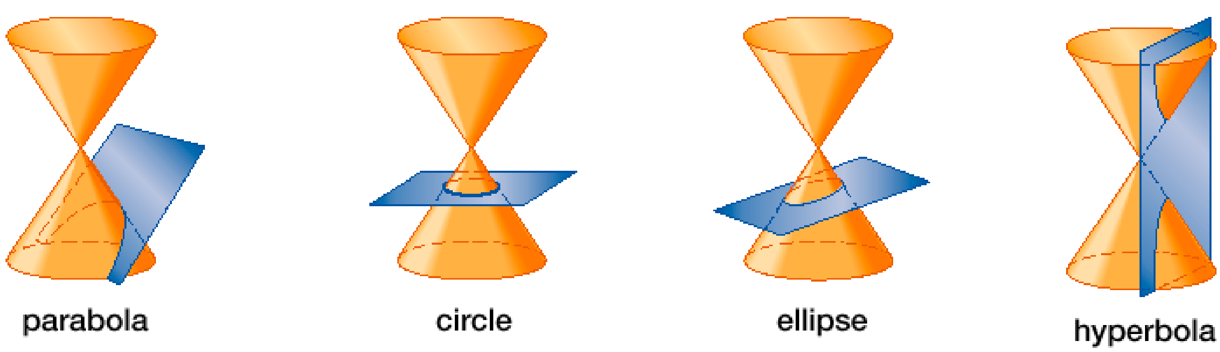
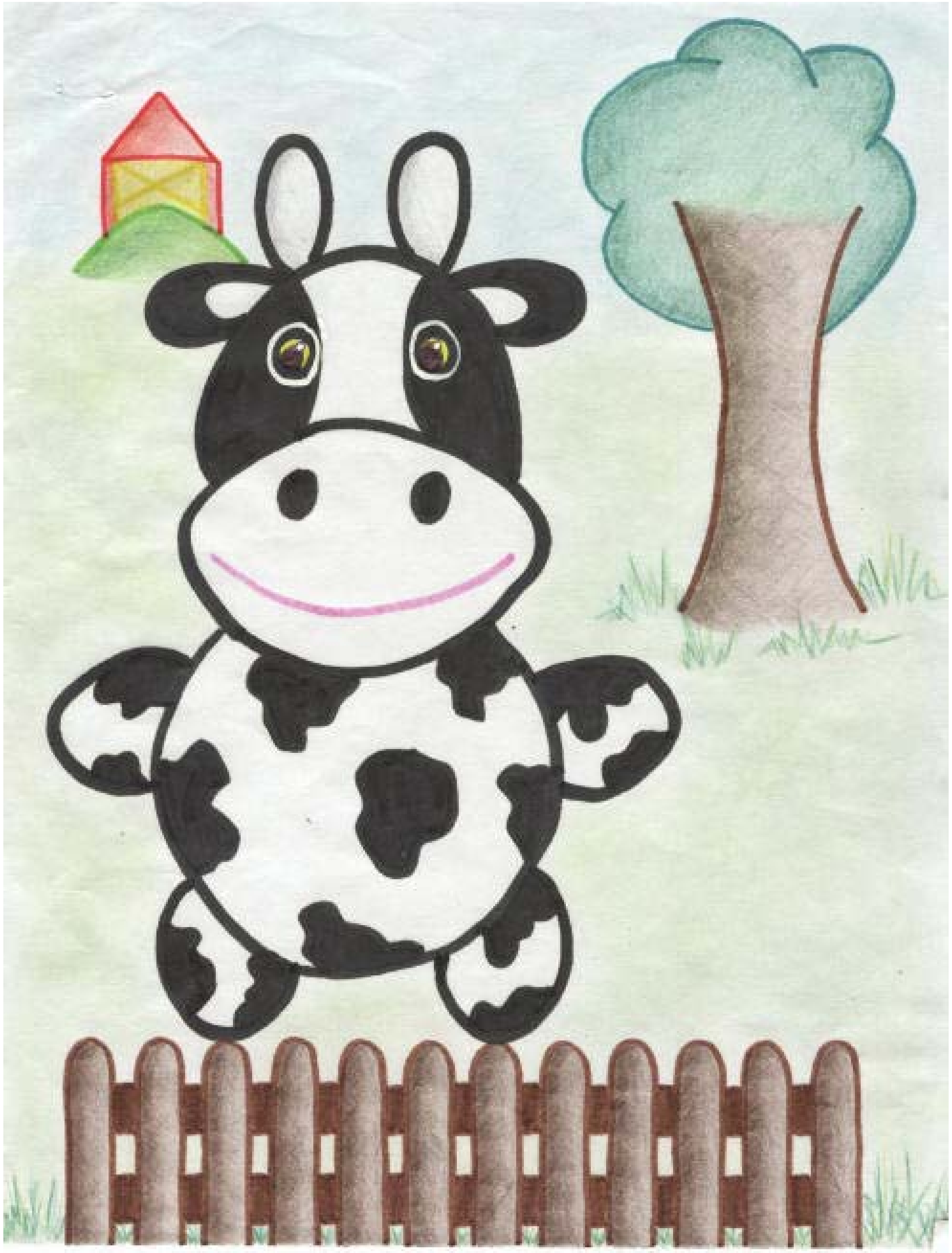


Source: Johanson, Terry. "Illuminations: Cutting Conics." *Illuminations: Welcome to Illuminations*. NCTM. Web. 10 Aug. 2010. <http://illuminations.nctm.org/LessonDetail.aspx?id=L792>.



Source: Cuevas, Gilbert J., Daniel Marks, Ruth M. Casey, Beatrice Moore-Harris, John A. Carter, Roger Day, and Linda M. Hayek. "Algebra Activity - Investigating Ellipses." *Algebra 2*. By Berchie W. Gordon-Holliday. New York: Glencoe/McGraw-Hill, 2005. 419. Print.

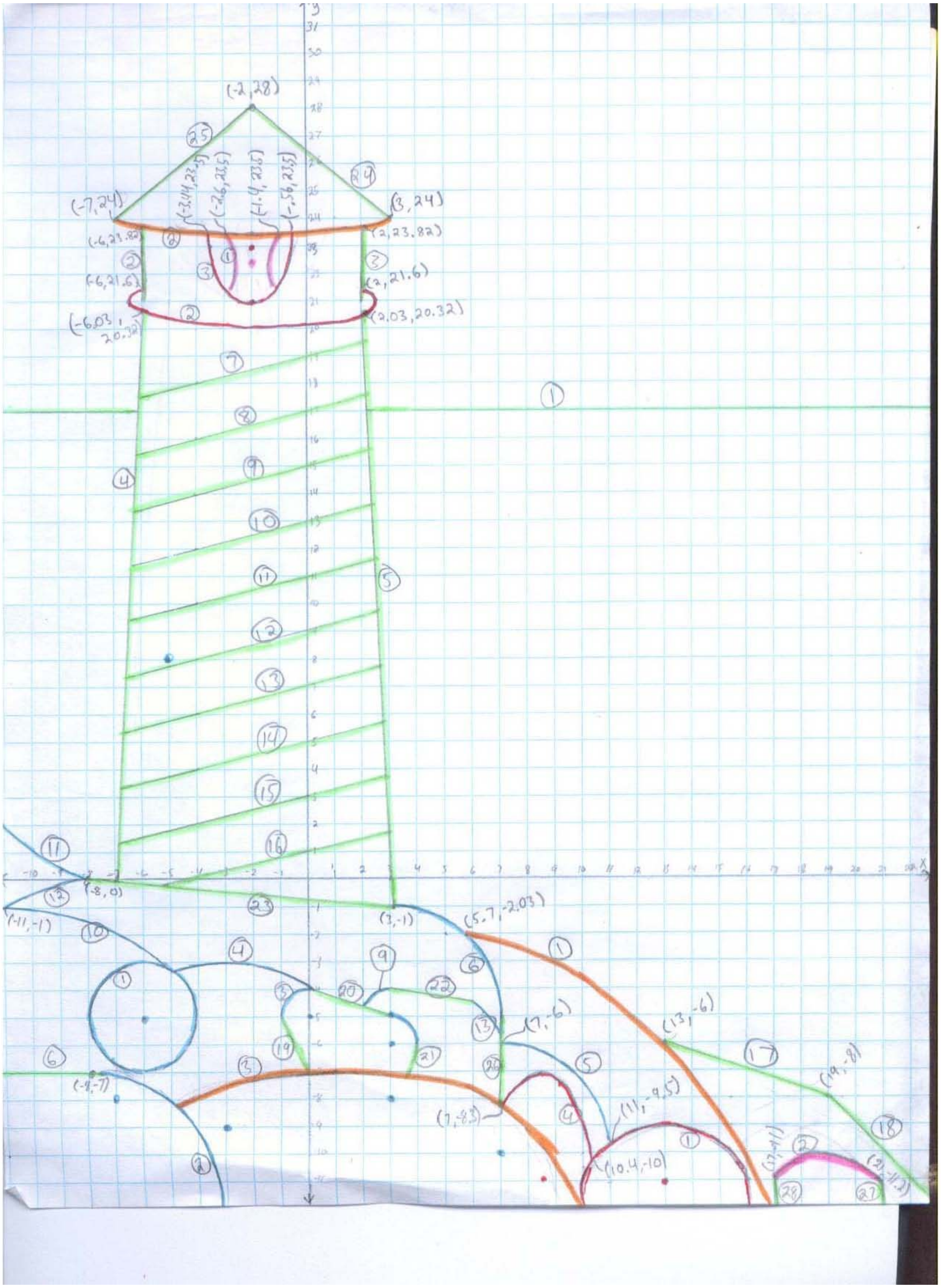
APPENDIX C: SAMPLES OF CONIC ART



Source: http://www.asfg.mx/highschool/Math/Projects/leticia-garcia/onceavo/conics2.htm

Source: http://www.asfg.mx/highschool/Math/Projects/leticia-garcia/onceavo/conics1.htm

Source: http://foothilltech.org/rvillano/lighthousecolor.jpg



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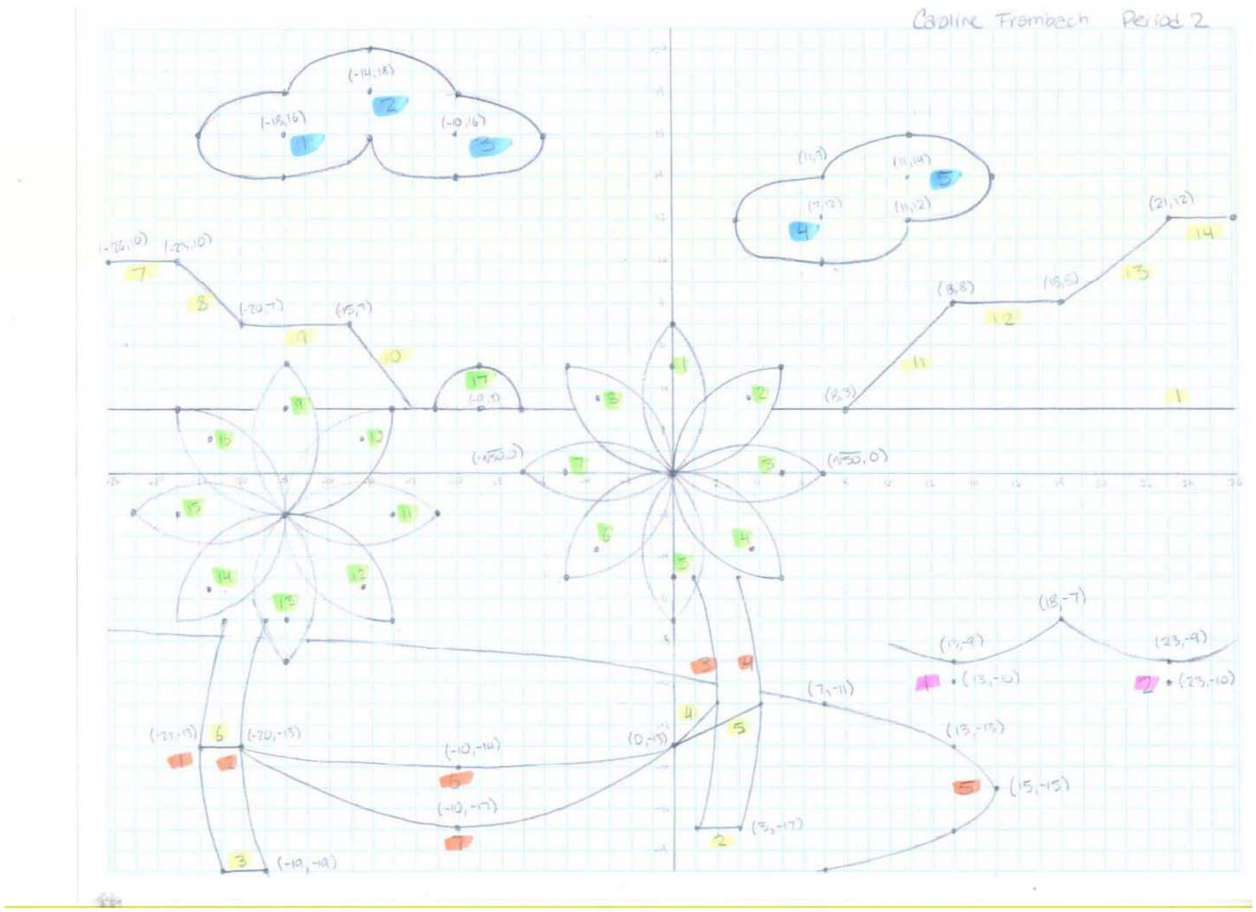

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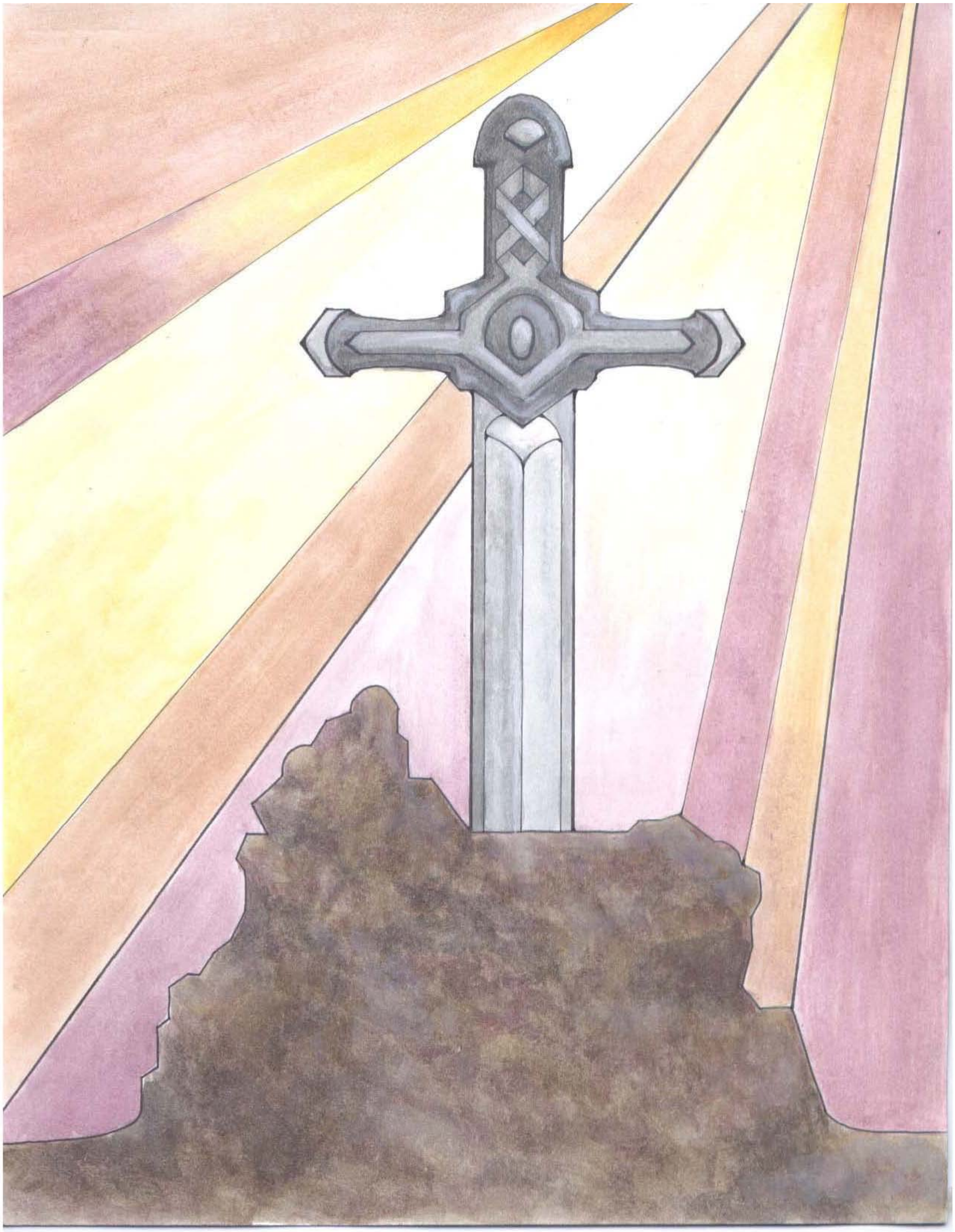

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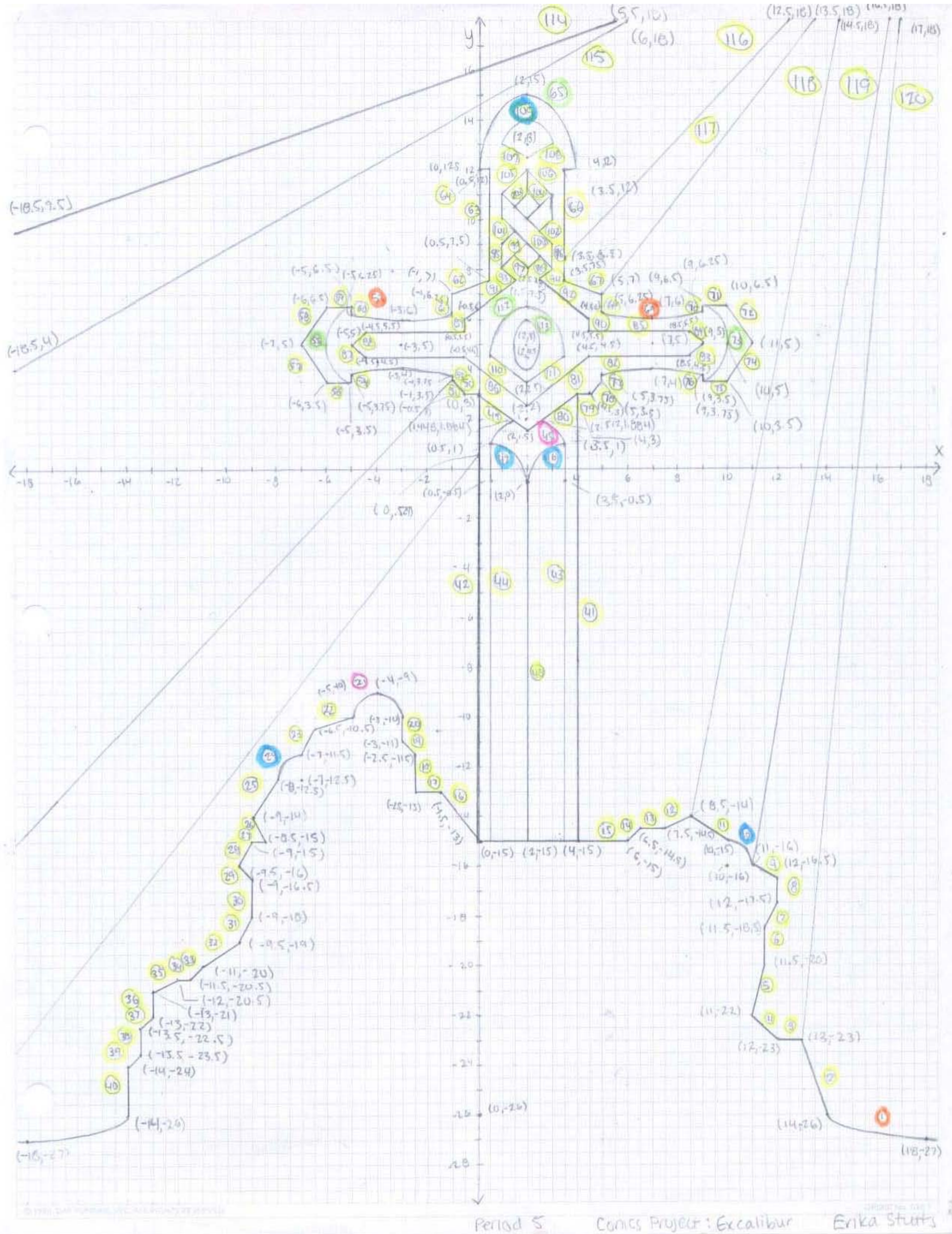


Source: http://foothilltech.org/rvillano/swordandthestonegraph.jpg

Conic Sections in Context

A Discovery-based, Multi-sensory Unit

By Elizabeth Richardson
Prepared for NMT ST 592 Independent Study Presentation

Statement of Purpose

The primary purpose of this unit is to examine conic sections and create connections between the geometric and algebraic definitions using relevant, student-centered instruction.

My Dilemma

- As a former student stated,
  "I just don't understand why we have to learn about this!"
- Perceived lack of relevancy frustrates and annoys the typical teenager
- In a traditional math class, primary mode of instruction is teacher-led

My Solution

- Create a unit that integrates explicit relevancy and focuses on student-centered instruction
- Overall goal is for students to be actively engaged in the process of discovery, reflection, and creation

Unit Plan

- Discovering Conic Sections through Technology
- Finding Meaning in the words of Mathematics
- Graphing with Sidewalk Chalk & Rope
- Clozing the Gap

Unit Plan

- Relating the Many Definitions of Conic Sections
- Creating with Conic Sections
- Conic Sections in Context Project
- Unit Test Review
- Unit Test

Measuring Success

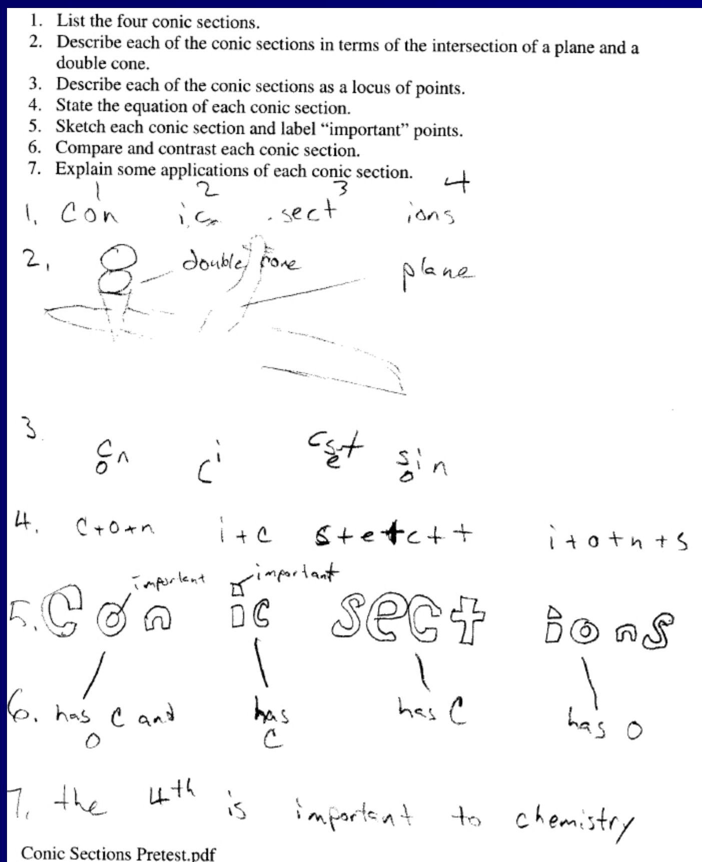
- Pre- and Post-test
- Daily Minute Paper
- Homework
- Projects
- Personal Progress Reflection
- Self-Assessment
- Unit Evaluation

Results

Comparison of Pre- and Post-test

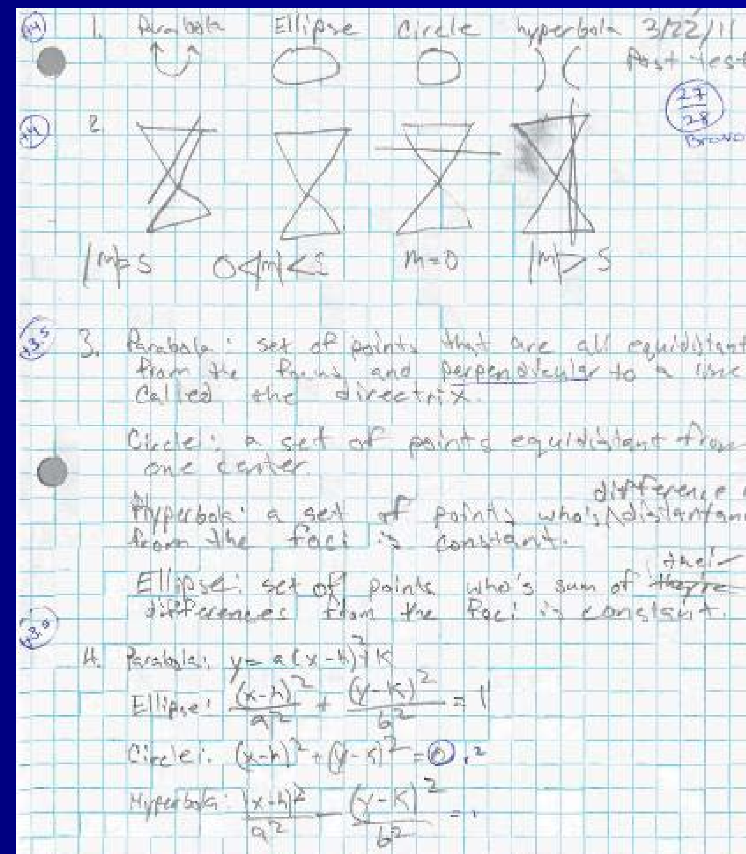
- Of the ninety-one students who completed the pre-test, three scored above 0 out of 28
- Following the completion of the Conic Sections in Context unit, student scores on the post-test averaged 23.125 out of 28 (~83%)
- Students from previous year averaged a 70.05% on the summative evaluation of conic sections

Results



1. List the four conic sections.
2. Describe each of the conic sections in terms of the intersection of a plane and a double cone.
3. Describe each of the conic sections as a locus of points.
4. State the equation of each conic section.
5. Sketch each conic section and label "important" points.
6. Compare and contrast each conic section.
7. Explain some applications of each conic section.

Conic Sections Pretest.pdf



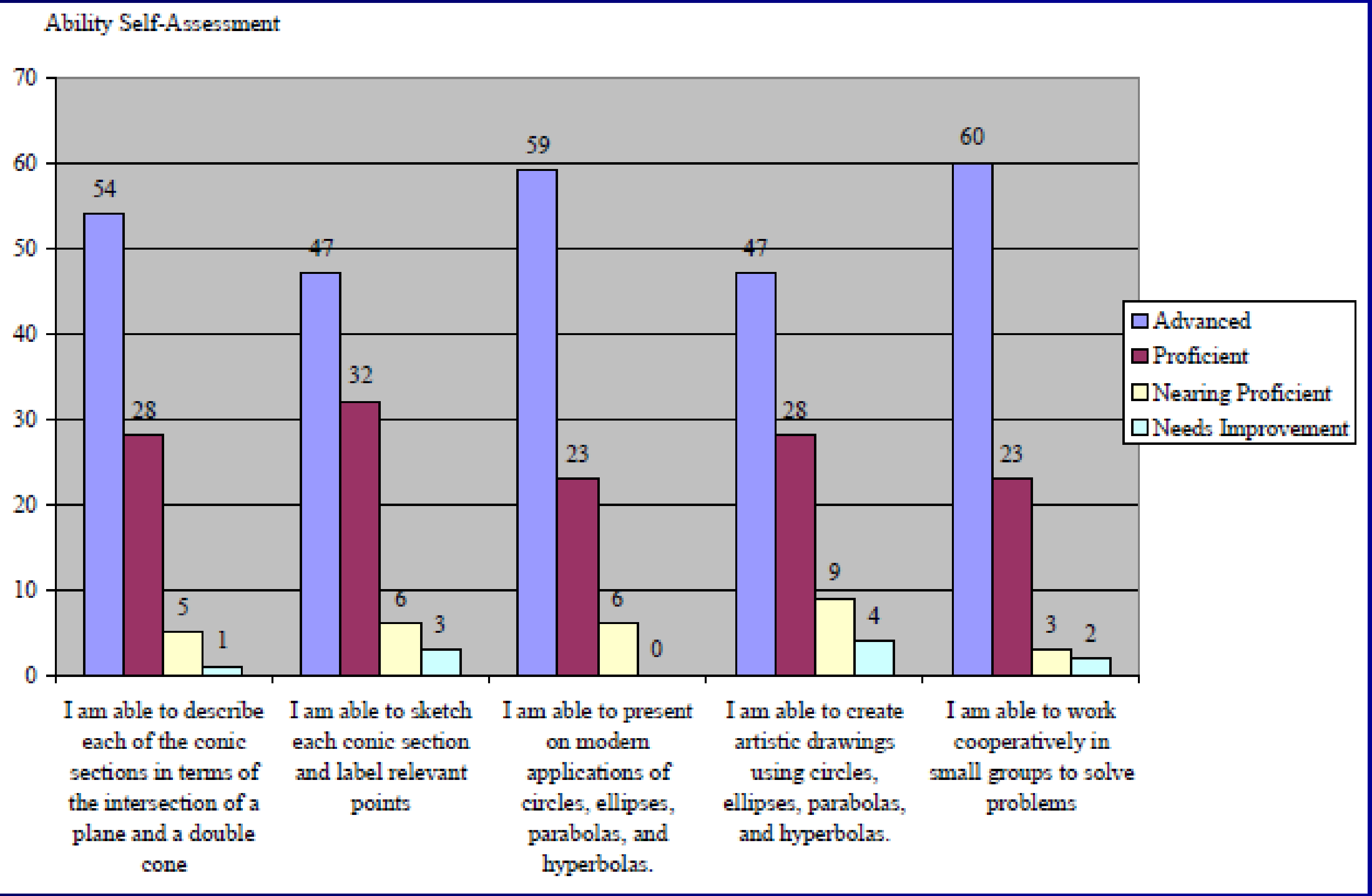
Results

- “Personally, I thought this unit would be a normal boring math section. I imagined countless equations and exercises and pointless attempts at explaining how this is going to affect you. However, this was not true. This unit expected you to think and explore.” (Justin)

- “It was a truly awesome experience to actually construct math so to speak... Math has never been my strong point, but during this section I really did feel confident. It gave me a feeling of accomplishment when I understood each concept.” (Valerie)

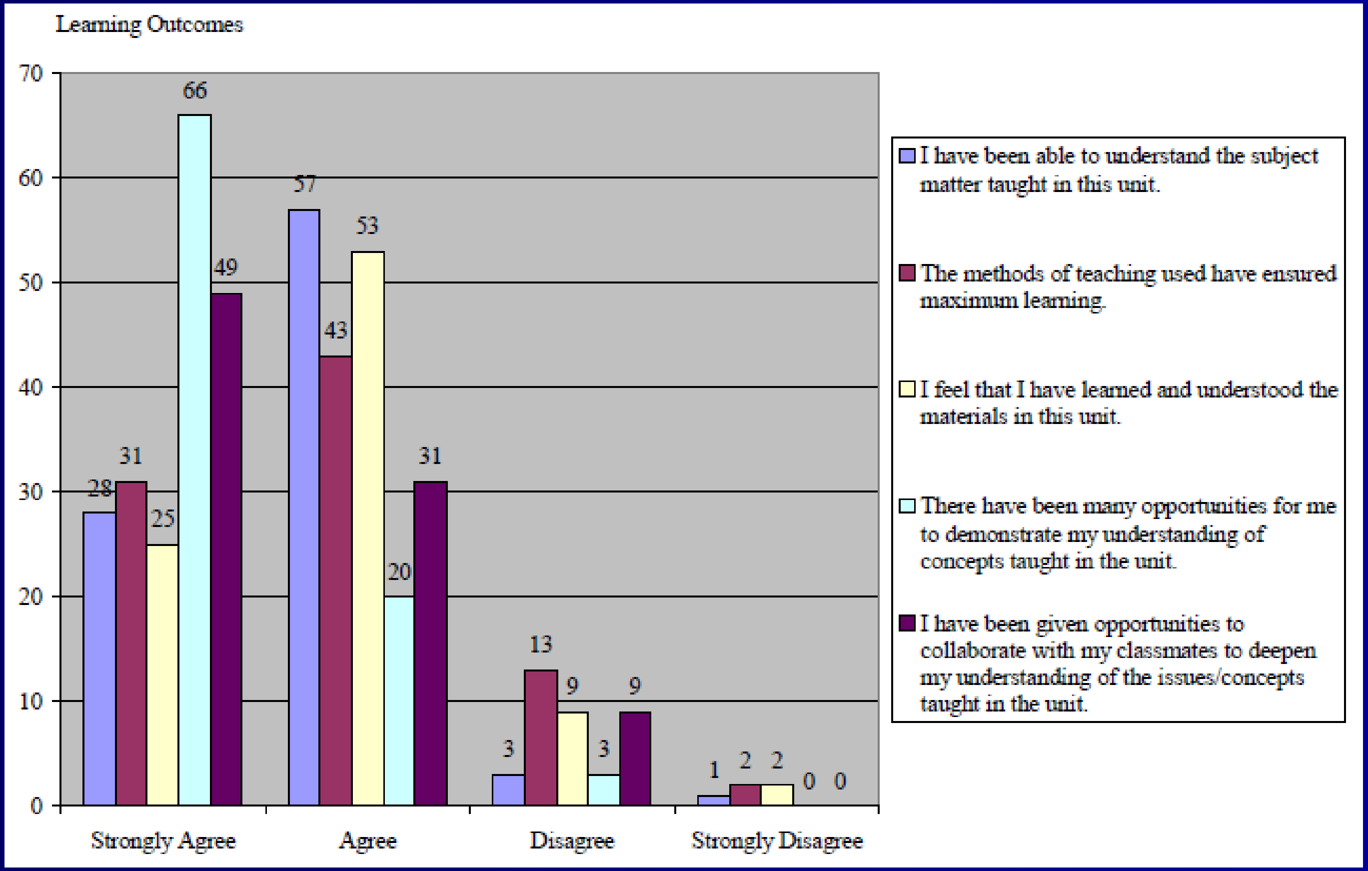
- “This unit not only taught me the material more efficiently, but it also taught me more about myself. I realize now that I can do well in math and enjoy it, as well as how I learn best.” (Brandon)

Results



Ability Self-Assessment
70
60
50
40
30
20
10
0
54
28
5
1
47
32
6
3
59
23
6
0
47
28
9
4
60
23
3
2
Advanced
Proficient
Nearing Proficient
Needs Improvement
I am able to describe each of the conic sections in terms of the intersection of a plane and a double cone
I am able to sketch each conic section and label relevant points
I am able to present on modern applications of circles, ellipses, parabolas, and hyperbolas.
I am able to create artistic drawings using circles, ellipses, parabolas, and hyperbolas.
I am able to work cooperatively in small groups to solve problems

Results



Learning Outcomes
70
60
50
40
30
20
10
0
28
31
25
66
49
57
43
53
20
31
3
13
9
3
9
1
2
2
0
0
Strongly Agree
Agree
Disagree
Strongly Disagree
I have been able to understand the subject matter taught in this unit.
The methods of teaching used have ensured maximum learning.
I feel that I have learned and understood the materials in this unit.
There have been many opportunities for me to demonstrate my understanding of concepts taught in the unit.
I have been given opportunities to collaborate with my classmates to deepen my understanding of the issues/concepts taught in the unit.

Revisions

- Start the unit with the webquest & continually refer to student-generated examples throughout the unit
- Focus on one conic section each lesson using a variety of approaches
- Create alternate lessons for absent students
- Differentiate assessment & allow student to choose

Conclusion

The overall goal of this unit was for students to be actively engaged in the process of discovery, reflection, and creation. I have no doubt this goal was met and am delighted to have it become a permanent component of the Algebra 2 curriculum of Los Alamos High School.

Q&A